

# Effect of temperature on dynamic behavior of cracked metallic and composite beam

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**Abstract.** Euler-Bernoulli beam model is used to analyze the thermal vibration of cracked metallic and laminated composite beam with fully clamped ends. An analytical solution technique is proposed to investigate the natural vibration of cracked beam subjected to axial thermal load. An open and stable crack is considered perpendicular to the beam axis. In this study, three different types of material such as steel, copper and aluminum and their composition are used to analyze the thermal effect on different modes of frequency. The effects of thermal stress on the vibration of cracked metallic and composite fully clamped beam are revealed. The results show that the effect of temperature is significant on the natural frequency of cracked metallic and composite beam.

**Keywords:** effect of temperature, natural frequency, crack, composite, beam.

## 1. Introduction

Effect of temperature is one of the important features for designing of structures efficiently. Unfortunately, thermal effect is ignored in most of the research which leads to incorrect results. A small change in temperature may cause a large variation in the behavior of beam vibration [1]. Thermal stresses are considered as mechanical stresses of the elements. It is the result of contraction or expansion of structural elements due to the applied thermal load. When an element is subjected to the rise of temperature it expands and when it loses temperature, it contracts. The expansion and contraction of the element due to the temperature is proportional to the change of temperature. This proportionality depends on the coefficient of linear thermal expansion of the material [2]. According to the theory of linear thermo-elasticity, the thermal strain is linearly added with the mechanical strain. That is why the equations of equilibrium and compatibility of thermal problems are the same as the elasticity problem. According to this assumption, many techniques that have been used to solve the elasticity problems are applicable to the thermo-elasticity problem as well [3].

Interest in multifunctional composite materials has increased rapidly in recent years. Laminated composite materials are also a very efficient type of composition. Laminated composite materials are formed by different layers of two or more materials that are effectively bonded together. It is a combination of a large number of fibers in a thin layer of matrix. Fibers in the lamina may be continuous or discontinuous, arranged in a specific direction or in a random orientation [4]. Lamination is a very useful technique to achieve more efficient material. Lamination can emphasize the properties of material such as strength, stiffness, low weight, corrosion resistance, thermal insulation, etc. [5-7].

The effect of thermal vibration on the cracked composite beam is very rare in the open literature. Therefore, thermal effects on macro mechanics also achieve substantial attention among researchers. Zhang et al. [8] studied the thermal effects on high-frequency vibration of beams. Their results showed that the spatial distributions and levels of energy density can be affected by the thermal effects. Cui and Hu [9] examined thermal buckling and natural vibration of the beam with an axial stick-slip-stop boundary. Lia and Zhang [10] investigated the thermal effect on

vibration and buckling analysis of thin plates with cracks. Their results revealed that the dynamic behavior of structural elements is significantly affected by thermal effects.

In this paper, the effects of temperature on the dynamic behavior of cracked metallic and composite beam are investigated. Our main concern is to compare the effectiveness of a fully clamped metallic and composite beam under crack and temperature. The governing differential equations with boundary and intermediate conditions are solved using analytical techniques. The effects of crack severity, crack position and beam length to height ratio on the frequencies of the heated beam are investigated. On the other hand, the effect of temperature on the vibration of the cracked composite beam is analyzed and compared with the metallic beam in detail.

## 2. Settings of the problem

A diagram of a cracked beam is illustrated in Fig. 1. The cross section of the beam is rectangular.  $L$  is the length and  $b$  is the width of the beam respectively. Beam is isotropic and homogeneous where  $\rho$  is the density of the metallic beam. Single crack is placed at the location of  $x = a$  of the beam. The crack is considered as stable crack whose depth is  $c$  and  $s$  is the ratio of crack height and beam height. Three different types of material (Aluminium, Copper, and Steel) are considered for the metallic beam as shown in Table 1.

A composite laminated material is also considered for the beam which is formed by three different materials such as aluminum, copper, and steel in a certain combination.

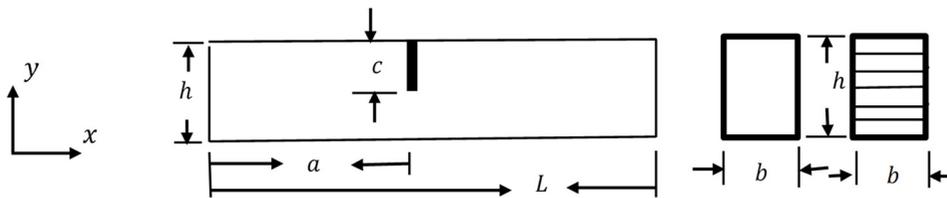


Fig. 1. Geometry of the problem

Table 1. Material properties

Material	Young's modulus (GPa)	Coefficient of thermal expansion ( $^{\circ}\text{C}$ )	Density ( $\text{kg}/\text{m}^3$ )
Aluminium	69	$24 \times 10^{-6}$	2780
Copper	117	$17 \times 10^{-6}$	8960
Steel	200	$13 \times 10^{-6}$	8050

## 3. Mathematical formulation

A beam is considered along the  $X$  axis where transverse deflection  $W(X, t)$  is described along the  $Y$  axis. According to the theory of Euler-Bernoulli for small deflection, the equation of motion expressed by the transverse deflection  $W(X, t)$  can be obtained as:

$$EI \frac{\partial^4 W}{\partial X^4} + \rho A \frac{\partial^2 W}{\partial t^2} + N_T \frac{\partial^2 W}{\partial X^2} = 0. \quad (1)$$

For separating the variable, this function  $W(X, t) = \bar{W} e^{i\omega c t}$  can be used. Then the Eq. (1) becomes:

$$EI \frac{\partial^4 \bar{W}}{\partial X^4} - \rho A \omega_c^2 + N_T \frac{\partial^2 \bar{W}}{\partial X^2} = 0. \quad (2)$$

Let us assume that the beam is homogenous having same temperature over its entire length. As a result of thermal expansion, an additional axial force  $N_T$  can be presented as:

$$N_T = -\alpha\theta EA. \tag{3}$$

$\alpha$  is the linear thermal extension coefficient,  $\theta$  is the temperature difference between the actual and initial temperature. Let us introduce dimensionless variables:

$$x = \frac{X}{L}, \quad w = \frac{\bar{W}}{L}, \quad \omega = \omega_c L^2 \sqrt{\frac{\rho h}{EI}}, \quad n = -\frac{N_T L^2}{EI} = 12 \alpha \theta \left(\frac{L}{h}\right)^2, \quad A = hb.$$

The Eq. (2) becomes:

$$\frac{\partial^4 w}{\partial x^4} - n \frac{\partial^2 w}{\partial x^2} - \omega^2 = 0. \tag{4}$$

Using this function  $w(x) = Ae^{i\lambda x}$ , we can write the characteristic equation as follow:

$$\lambda^4 + n\lambda^2 - \omega^2 = 0. \tag{5}$$

Roots of the equation can be presented as follow:

$$\lambda_{1,2} = \pm i \sqrt{\frac{\sqrt{n^2 + 4\omega^2} + n}{2}} = \pm i\alpha, \quad \lambda_{3,4} = \pm \sqrt{\frac{\sqrt{n^2 + 4\omega^2} - n}{2}} = \pm\beta.$$

Solutions of the equation are:

$$w = A_1 \sinh(\alpha x) + A_2 \cosh(\alpha x) + A_3 \sin(\beta x) + A_4 \cos(\beta x), \quad 0 < x < a, \tag{6}$$

$$w = A_5 \sinh(\alpha x) + A_6 \cosh(\alpha x) + A_7 \sin(\beta x) + A_8 \cos(\beta x), \quad a < x < L. \tag{7}$$

Boundary conditions for the fully clamped beam as follow:

$$w(0) = 0, \quad w(L) = 0, \quad w'(0) = 0, \quad w'(L) = 0. \tag{8}$$

These four boundary conditions and other four intermediate conditions for crack are used to solve the Eqs. (6) and (7). For crack, the stress intensity factor for the single edge notched tension is presented as follow (see [11]):

$$C = \frac{\sqrt{2 \tan(\pi c/2h)}}{\cos(\pi c/2h)} \left[ 0.752 + 2.02 \left(\frac{c}{h}\right) + 0.37 \left(1 - \sin \frac{\pi c}{2h}\right)^3 \right]. \tag{9}$$

Using the intensity factor intermediate conditions for the crack can be defined as below:

$$w_i(x_i) = w_{i+1}(x_i), \quad w_i''(x_i) = w_{i+1}''(x_i), \\ w_{i+1}'(x_i) - w_i'(x_i) = C w_i''(x_i), \quad w_i'''(x_i) = w_{i+1}'''(x_i). \tag{10}$$

In composite beam, the Young's modulus of composite system can be written as:

$$E = E_s V_s + E_c V_c + E_a V_a, \quad \epsilon = \epsilon_s = \epsilon_c = \epsilon_a. \tag{11}$$

Thermal expansion coefficient of composite material can be presented as:

$$\alpha = \frac{E_s \alpha_s V_s + E_a \alpha_a V_a + E_c \alpha_c V_c}{E_s V_s + E_a V_a + E_c V_c}. \tag{12}$$

#### 4. Results and discussion

The temperature effect on the vibration of metallic and composite beams is significant. To develop a reliable model to analyse thermal effect on metallic and composite beam are also very essential. In this section, the outcomes of this analysis are presented with graphs and tables. Three different materials such as steel, copper, aluminium and their composite are used in this analysis. In composite, these three materials such as steel, copper, and aluminium are used as a ratio of 40 %, 30 %, and 30 % respectively. The results of this analysis show good accordance with the papers of other researcher in open literature.

**Table 2.** Natural frequency of solid beam with clamped ends

$n = 1$	Frequency in different modes					
	1	2	3	4	5	6
Current work	22.095	61.295	120.49	199.44	298.09	417.19
Jiang and Wang [12]	21.736	57.967	107.74	166.61	231.10	298.71

Table 2 illustrates the dimensionless natural frequency of fully clamped solid beam. Current research data show good agreement with data of other researcher [12] in lower mode of frequency. In current work, Euler-Bernoulli theory is used. On the other hand, Jiang and Wang [12] used nonlocal theory of elasticity with nonlocal parameter value 0.05. That is why, there are some differences in both data sets in higher modes of frequency.

**Table 3.** Natural frequency of solid beam for different axial loads

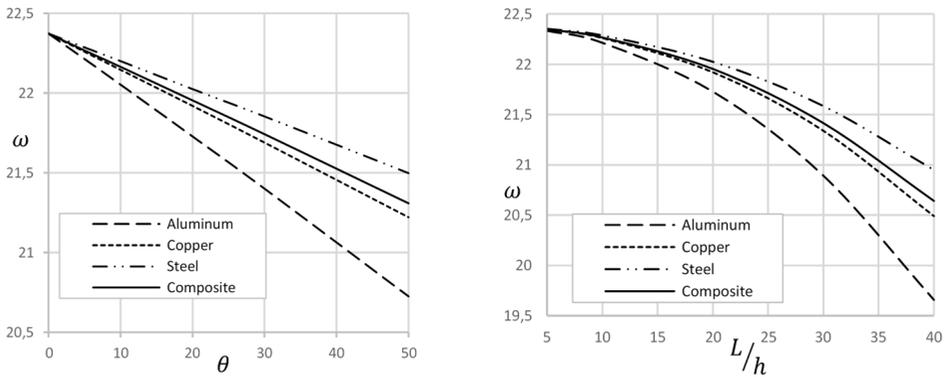
$n$	Frequency in different modes					
	1	2	3	4	5	6
0	22.37383	61.67188	120.9063	199.8543	298.5484	417.6172
2	21.81445	60.91797	120.0859	198.9844	297.6367	416.7578
4	21.24023	60.16211	119.2523	198.1406	296.7656	415.9063
8	20.04102	58.60547	117.5859	196.3945	294.9844	414.1328
16	17.36621	55.35547	114.1641	192.8613	291.3594	410.6563
32	9.87793	48.15234	106.998	185.6094	284.0156	403.7109

Table 3 depicts the natural frequency of solid beam under dimensionless axial loads. Natural frequency decreases with increase of axial load. Axial load is very effective on the 1st mode of frequency instead of higher mode of frequency.

**Table 4.** Natural frequency of composite beam for different temperatures

$L/h=20$	Aluminum (30%), Copper (30%), Steel (40%)			Aluminum (40%), Copper (30%), Steel (30%)			Aluminum (30%), Copper (40%), Steel (30%)		
	1	2	3	1	2	3	1	2	3
$\theta$									
0	22.3730	61.6738	120.898	22.3744	61.6736	120.898	22.373	61.6738	120.898
5	22.2676	61.5352	120.742	22.2637	61.5273	120.742	22.2676	61.5273	120.742
10	22.1660	61.3867	120.602	22.1543	61.377	120.572	22.1582	61.3789	120.586
20	21.9551	61.1055	120.289	21.929	61.0723	120.246	21.9395	61.0898	120.258
30	21.7402	60.8242	119.977	21.7046	60.7695	119.914	21.7207	60.793	119.945
40	21.5254	60.5352	119.668	21.4746	60.4727	119.586	21.498	60.4961	119.617
50	21.3076	60.2461	119.345	21.2441	60.1602	119.254	21.2715	60.1992	119.305

Table 4 reveals the effect of temperature on natural frequency of composite beam. Three different types of materials such as steel, copper and aluminum are used in these composites. Three different composition ratios of these materials are used to create three different composites. Table data show that increase of temperature decreases frequency. On the other hand, ratio of these compositions is not very significant on natural frequency.

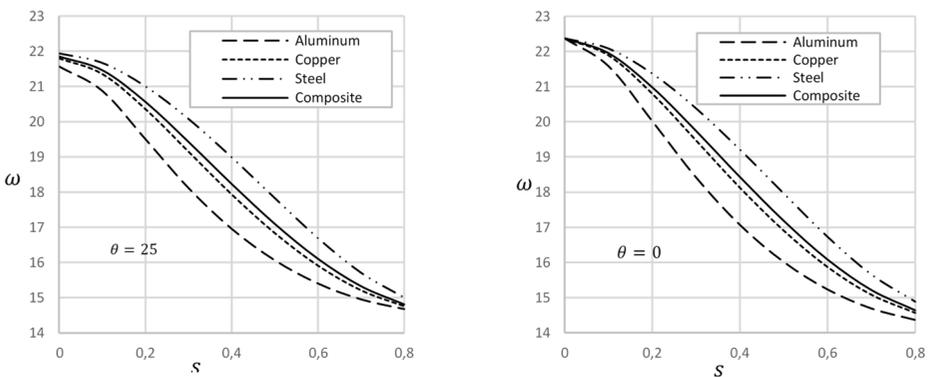


**Fig. 2.** Frequency versus change of temperature and length to height ratio

Fig. 2 illustrates the effect of temperature and length to height ratio on the natural frequency of beam for different materials. Natural frequency decreases with increase of temperature and the length to height ratio. Temperature increases axial tension that decreases natural frequency. Similarly, increase of length to height ratio decreases height and cross sectional area of beam that increases axial stress. Increase of axial stress decreases natural frequency. Composite material shows the effective results considering the behavior of other materials.

Fig. 3 displays the effect of crack depth ratio on the natural frequency presence and absence of temperature. This figure depicts that increase of crack depth ratio decreases natural frequency. On the absence of temperature, the effect of crack depth ratio is more significant than the presence of that. High temperature expands the length of the beam and reduces the width of the crack that decreases slightly the effect of crack on frequency.

Fig. 4 demonstrates the effect of temperature on the frequency of cracked beams with different crack-depth ratio. At the low crack depth ratio, frequency decreases with the increase of temperature. On the other hand, at the high crack depth ratio, frequency increases with the increase of temperature. Increase of temperature expands the length of the beam and decreases the width of the crack that enhances the frequency of the beam. Fig. 5 describes the effect of crack location on the natural frequency of beam on presence and absence of temperature. Crack position at the ends and at the mid of the beam show lower frequency. On the other hand, in between crack position shows higher frequency of the beam. Frequency of crack beam with temperature is more significant than the crack without temperature.



**Fig. 3.** Frequency versus crack depth ratio with and without temperature

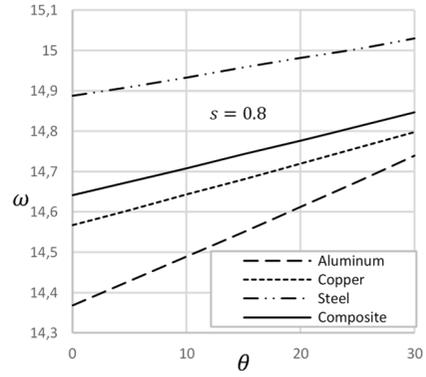
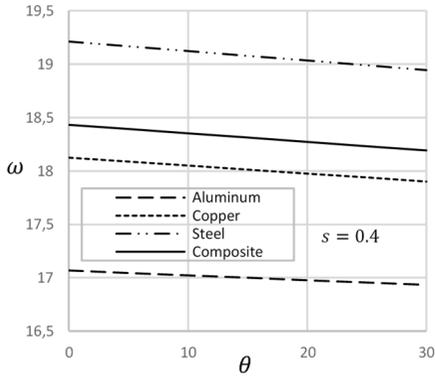


Fig. 4. Frequency versus change of temperature for different crack depth ratios

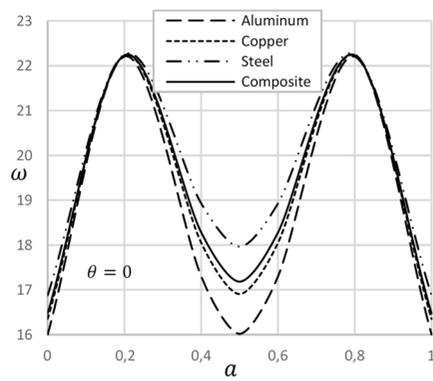
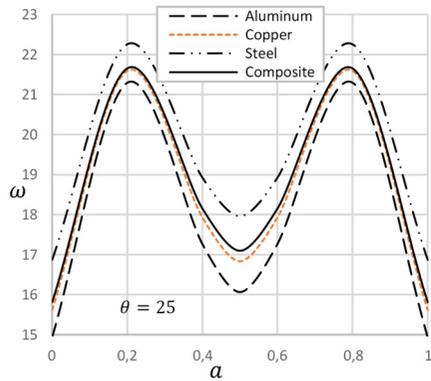


Fig. 5. Frequency versus crack location with and without temperature

## 5. Conclusions

In this study, Euler-Bernoulli beam theory is employed to develop an analytical technique for analyzing thermal effect on dynamic behavior of cracked metallic and composite beam. The results are presented for three different types of material and their composite. The results of this investigation showed that the effect of thermal vibration depends on length to height ratio of the beam. The effect of temperature is more significant in first mode of frequency. In higher mode of frequency, the temperature effect is less significant. Under thermal load composite beam is very effective than metallic beam.

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