

1.1. Mathematical model

In the development of the mathematical model it is assumed that the relative material removal rate is proportional to the relative velocity of the piece being ground and the polishing pad (Preston's law [3]).

Introducing the notation:

$$O_1B = b, \quad AB = c, \quad AO_2 = d, \quad O_2O_3 = R_0, \quad O_2O_4 = R_1, \quad O_2O_1 = f, \quad \psi = \omega t,$$

and making a series of simple transformations, we obtain the formula for calculating the velocity V_r of the point M on the holder relative to the polishing pad in the form:

$$V_r = R_0 \sqrt{[\rho_2(\omega_0 - \dot{\varphi})\sin\gamma + \rho_1\omega_0\sin\mu]^2 + [\rho\dot{\varphi} + \rho_2(\dot{\varphi} - \omega_0)\cos\gamma - \rho_1\omega_0\cos\mu]^2}, \quad (1)$$

where:

$$\begin{aligned} \dot{\varphi} &= \omega [f^{-1}b\cos(\varphi - \psi) - d^{-1}b\cos\psi][f^{-1}b\cos(\varphi - \psi) - \sin\varphi]^{-1}, \\ \Phi(t) &= (G - \Phi_1(t))/2d\sqrt{f^2 + b^2 - 2fb\sin\psi} = \sin(\varphi - \vartheta), \\ G &= f^2 + d^2 + b^2 - c^2, \quad \Phi_1(t) = 2fb\sin\psi, \quad \cos\varphi_0 = \frac{f^2 - c^2 + d^2}{2fd}, \\ \rho &= \frac{R}{R_0}, \quad \rho_1 = \sqrt{1 + \rho(\rho - 2\cos(\varphi - \varphi_0))}, \quad \rho_2 = \frac{r}{R_0}, \\ \operatorname{tg}\vartheta &= \frac{f - b\sin\psi}{b\cos\psi}, \quad \operatorname{tg}\mu = -\sin\varphi_0(\rho - \cos(\varphi - \varphi_0))^{-1}. \end{aligned}$$

It follows from Preston's law [3] that the relative rate of material removal at different points of the wafers on the holder $S(r, \gamma) = S(\rho_2, \gamma)$ is determined from the following Eq. (2):

$$S = \int_0^{2\pi/\omega} V_r(t) dt. \quad (2)$$

Fig. 2 shows for $f = 22.7$; $c = 22.5$; $d = 3.5$; $R_0 = 40$; $b = 0.4$. in three-dimensional space $[\gamma, \rho, V/R_0(S)]$ the results of the calculations of the relative value of material removal on the wafers being ground depending on the coordinates of the wafer points: the angle γ , the radius ρ , ω at $\omega_0 = 0.5c^{-1}$ for the mode of the holder fixed on its axis, using the MATLAB package. Let us now consider in more detail the process of grinding in the free mode of the holder. In this case, the holder's rotation is determined by the action of the moment M_1 of the frictional forces on the holder from the pad and the friction torque on the axis of the holder.

To determine M_1 , we choose a coordinate system $x'y'$ parallel to the system xy with the origin that coincides with the center of the holder. In this system, the inertial forces acting on small parts of the holder are equal to $\Delta\mathbf{F} = -\Delta m\mathbf{W}_0$ where Δm is a part of the mass, and \mathbf{W}_0 is the acceleration of the center of the holder. These forces do not generate torque, since the holder is disk-shaped and the wafers being ground are fixed, as a rule, symmetrically without changing the position of the center of gravity that coincides with the center of the disk. Thus we conclude that the moment M_1 is only determined by the action of the friction forces between the holder (the wafers on the holder) and the polishing pad.

A detailed study of the friction forces associated with the process of polishing is a separate large task. Therefore, to simplify, we assume that the frictional force acting on the element of the holder with the area dS is equal to:

$$d\mathbf{F}_Z = -\chi \mathbf{v}_r ds. \quad (3)$$

The admissibility of this representation of the frictional forces is based on the similarity of the abrasive with lubricant to a high viscosity liquid. Constant χ is proportional to its viscosity. Then we can write Eq. (4):

$$d\mathbf{M}_1 = [\mathbf{r} \cdot d\mathbf{F}_Z] = -\chi [\mathbf{r} \times \mathbf{v}_r] ds. \quad (4)$$

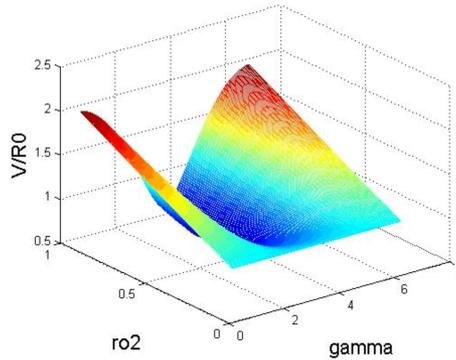


Fig. 2. The dependence of the relative rate of material removal S on the coordinate points of the wafer

Going to the coordinate system ξ, ζ, η fixed relative to the holder (the axis ξ is directed along O_4W , the axis ζ – along O_4U , the axes ξ, ζ, η form a right-handed space) and calculating the vector value $[\mathbf{r} \times \mathbf{v}_r]$ as:

$$[\mathbf{r} \mathbf{v}_r] = r(\cos(\gamma)v_{r\xi}[\mathbf{e}_\xi \mathbf{e}_\zeta] + \sin(\gamma)v_{r\zeta}[\mathbf{e}_\xi \mathbf{e}_\zeta]) = r(\cos(\gamma)v_{r\zeta} - \sin(\gamma)v_{r\xi})\mathbf{e}_\eta,$$

it is possible to compute the moment relative to the axis η in the following way:

$$M_\eta = -\chi \int_0^{r_0} \int_0^{2\pi} r^2 [\cos^2(\gamma)(\omega_0 R + (\omega_3 - \omega_0)r \cos(\gamma) - \omega_0 r \cos(\mu)) - \sin(\gamma)((\omega_3 - \omega_0)r \sin(\gamma) - \omega_0 r \sin(\mu))] dr d\gamma = -\frac{\chi \pi r_0^4 (\omega_3 - \omega_0)}{2}, \quad (5)$$

where ω_3 is the angular speed of the holder in the coordinate system x and y . Here, r_0 is the radius of the holder. Thus, in the chosen coordinate system we have:

$$J \frac{d\omega_3}{dt} = -\frac{\chi \pi r_0^4 (\omega_3 - \omega_0)}{2} - M_Z, \quad (6)$$

where J is the moment of inertia of the holder with the wafers. Assume further that the force of friction on the axis is the Coulomb-Amontons friction and represent the moment in the following form:

$$M_Z = B \text{sign}(\omega_2), \quad \omega_2 \neq 0, \quad M_Z \in [-B, B], \quad \omega_2 = 0.$$

Since $\omega_3 = \omega_1 + \omega_2$, where ω_2 is the velocity of the holder relative to its axis, and ω_1 is the angular velocity of the holder drive piece, which can be considered for the above scheme approximately equal to $\omega_1 = A \sin(\omega t + \varphi_0)$, ω_0 is the polishing pad velocity, the Eq. (6) can be rewritten as Eq. (7):

$$\frac{d\omega_2}{dt} + \lambda\omega_2 = -A(\omega\cos(\omega t + \varphi_0) + \lambda\sin(\omega t + \varphi_0)) + \lambda\omega_0 - \frac{M_z(\omega_2)}{J}, \quad \lambda = \frac{\chi\pi r_0^4}{2J}. \quad (7)$$

Introducing the dimensionless time $\tau = \omega t + \varphi_0 - \Phi$, coordinate $x = (\omega_2 - \omega_0)/\omega_0$ and parameters $k = \lambda/\omega$, $F = A\sqrt{\lambda^2 + \omega^2}/(\omega\omega_0)$, $a = B/(J\omega_0\omega)$, Eq. (7) can be rewritten as follows Eq. (8):

$$\dot{x} + kx = -F\cos\tau - \text{asign}(x + 1), \quad \text{tg}\Phi = \frac{\lambda}{\omega}. \quad (8)$$

The phase space of Eq. (8) is two-dimensional in the coordinates (τ, x) . It has a straight line Γ ($x = -1$), which divides the plane into three subspaces $X_+(x > -1)$, $X_-(x < -1)$ and Γ ($x = -1$). The motion of the image point in said subspaces is described by the Eq. (9):

$$\dot{x} + kx = -F\cos\tau - \text{asign}(x + 1), \quad x \neq -1, \quad |k + F\cos\tau| \leq a, \quad x = -1. \quad (9)$$

Note that the motion of the image point in the subspace Γ ($x = -1$) takes place in time intervals $\Delta_i \in [\tau_i, \tau_{i+1}]$ corresponding to the junction of the phase trajectories coming from the subspaces X_-, X_+ .

Fig. 3 (a, b) shows the phase trajectories for different parameter values $a = 9$ (Fig. 3(a)) and $a = 7$ (Fig. 3(b)), respectively, at $k = 0.71$, $F = 3.2$.

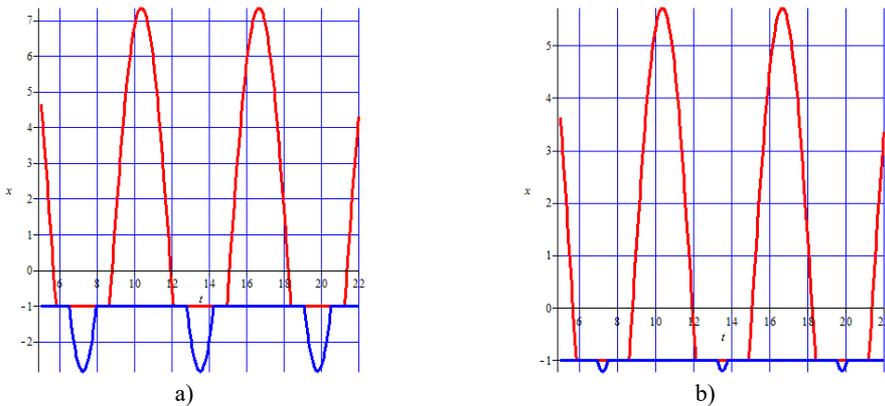


Fig. 3. Phase trajectories in the plane τ, x

What all the parameters have in common is the presence of parameter intervals Δ_i of the holder stops, which increase with a rise of the coefficient of static friction. The rotation with the stopping of the holder begins only from some values of a (see Fig. 3(b)). With the increase of the parameter k the intervals are shifted to the region of the shorter times, and the oscillation amplitude decreases. In the ideal case, when we can neglect friction on the axis of the holder ($M_2 = 0$), it follows from Eq. (6) that with the obvious relation $(\chi r_0^4 \pi / 2) / (J \omega_0) > 0$ we will obtain $\lim_{t \rightarrow \infty} \omega_3(t) = \omega_0$. This means that the angular velocity of the holder ω_2 relative to its axis is $\omega_2 = \omega_0 - \omega_1(t)$. Note that for the chosen values of the parameters of the mechanism the relation $\omega_1 = \dot{\varphi} \approx 0.25\sin(\omega t + \varphi_0)$ holds true and hence $\omega_2 > 0$. Assuming $M_2 \neq 0$ and that it can be approximated by the force of friction obeying the Coulomb-Amontons law, it follows from Eq. (6) that $\lim_{t \rightarrow \infty} \omega_3(t) = \omega_0 - \Delta\omega$, where $\Delta\omega$ is a small constant, which is equal to $\Delta\omega = 2|M_2|_{\max} / (\chi\pi r_0^4)$. In this case, for the steady state mode, instead of the relationship Eq. (1) we will have the following Eq. (10):

$$V_r = R_0 \sqrt{[\rho_2 \Delta \omega \sin \gamma + \rho_1 \omega_0 \sin \mu]^2 + [\rho \dot{\varphi} + \rho_2 \Delta \omega \cos \gamma - \rho_1 \omega_0 \cos \mu]^2}. \quad (10)$$

2. Conclusions

Note that according to Eq. (10):

- In the ideal case ($M_2 = 0$) there is no dependence of the relative velocity V_r from the position of the point on the holder ρ_2 , that is, the removal of the material on the grinding disc will be uniform across the surface;
- The friction on the axis of the holder leads to a weak dependence V_r from ρ_2 and thus to the non-uniform removal of the material;
- The dynamics of the mechanism is determined by the ratio of the friction torque on the axis and in the contact of the wafers with the polishing pad;
- There are three modes of the mechanism motion: free rotation, rotation with periodic stops of the holder; the holder does not rotate on its axis.

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References

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