

The dynamics of a nonautonomous oscillator with friction memory

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Abstract. In this article, an oscillator is considered in which one of its component moves periodically and for which static friction is taken to be time-dependent. The dynamics of the oscillator are analyzed using Poincare map to find shifts between periodic and chaotic motion with the change of the model parameters (amplitude and frequency of velocity of the periodic motion and the coefficient of static friction as a function of the time of stationary contact).

Keywords: mathematical model, Poincare map, bifurcation diagram, time-dependent static friction, chaos.

1. Introduction

A. Y. Ishlinskiy and I. V. Kregelskiy [1] presented a hypothesis that the coefficient of static friction is not a constant, but a monotonously increasing function of the time of the contact between two bodies. After a substantial delay, it attracted the attention of scientists working on systems with friction [2-4]. It has been shown, that systems with time-dependent static friction can display different types of dynamic behavior, including complex periodic motion and chaos. Similar systems to the one studied here have also been studied in other works (e.g., [5-7]), but they did not use time-dependent static friction and used the classic model of static friction instead.

1.1. Mathematical model

The model considered in this article is depicted in Fig. 1(a).



Fig. 1. Physical model of a vibration system

The model consists of a mass m , that rides on a moving belt with dry friction. The belt moves with periodically changing velocity $V(t)$. The mass is attached to inertial space by a spring with stiffness k . The coefficient of static friction, in accordance with the hypothesis of Ishlinskiy A. Yi., Kragelskiyi I. V. [1], is considered to be a continuous monotonically increasing function of the time t_k of stationary contact (stick motion of the mass and the belt) as depicted in Fig. 1(b). The Coulomb model is used for non-static dry friction.

The motion of the mass in the model is described by the Eq. (1), (2):

$$m\dot{x} = -kx - f_*P\text{sign}(\dot{x} - V(t)), \quad \dot{x} \neq V(t), \quad (1)$$

$$|kx + m\dot{V}(t)| \leq f(t_k)P, \quad \dot{x} = V(t). \quad (2)$$

Eq. (1) describes the slip motion of the mass with non-static dry friction with coefficient f_* .

Eq. (2) describes the condition for the stick motion. $f_n(t_k)$ is the coefficient of static friction as a function of the stick motion time t_k . Note that when $V(t) = const$, the Eq. (1) takes the form of the corresponding equation in [4].

With dimensionless time $\tau = t\omega_0$, coordinate $\xi = xk/f_*P$ and parameters $\omega_0 = \sqrt{k/m}$, $\theta(\tau) = V\sqrt{km}/f_*P$, the Eq. (1), (2) will take the form:

$$\ddot{\xi} + \xi = -\text{sign}(\dot{\xi} - \theta(\tau)), \quad \dot{\xi} \neq \theta(\tau), \tag{3}$$

$$|\dot{\xi} + \dot{\theta}(\tau)| \leq 1 + \varepsilon(\tau_k), \quad \dot{\xi} = \theta(\tau), \tag{4}$$

where $\varepsilon(\tau_k) = (f_n(\tau_k) - f_*)f_*^{-1}$ – dimensionless coefficient of static friction).

1.2. Structure of the phase space

Since the system is non-autonomous and can be described by second-order differential equation, its state is a triplet $\{\xi, \dot{\xi}, \tau\}$ (Fig. 2), and its phase space is accordingly 3-dimensional. The phase space is split into 2 half-spaces $\Phi_1(\xi, \dot{\xi} > \theta, \tau)$ and $\Phi_2(\xi, \dot{\xi} < \theta, \tau)$ by the surface $\Pi(\dot{\xi} = \theta(\tau))$.

The shape of phase trajectories in each half-space is described by the Eq. (5), (6):

$$\ddot{\xi} + \xi = -1, \quad \dot{\xi} > \theta(\tau), \tag{5}$$

$$\ddot{\xi} + \xi = +1, \quad \dot{\xi} < \theta(\tau). \tag{6}$$

It can be shown, that on the surface Π there is a strip of stick motions Π_C bounded by curves Γ_1 and Γ_2 :

$$\Gamma_1: \begin{cases} \xi = 1 - \dot{\theta}(\tau), \\ \dot{\xi} = \theta(\tau), \end{cases} \quad \Gamma_2: \begin{cases} \xi = -1 - \dot{\theta}(\tau), \\ \dot{\xi} = \theta(\tau). \end{cases} \tag{7}$$

On the strip Π_C a phase trajectory is described by Eq. (8):

$$\xi(\tau) = \int_{\tau_n}^{\tau} \theta(\eta) d\eta + \xi_n, \quad \{\xi_n, \tau_n\} \in \Pi_C. \tag{8}$$

An example of a phase space trajectory with stick motion intervals is shown in Fig. 3.

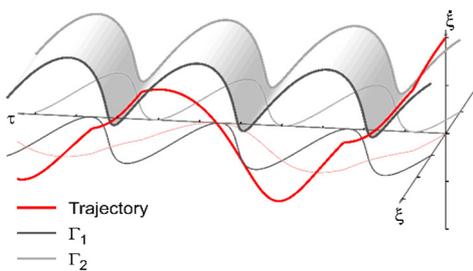


Fig. 2. Phase space S

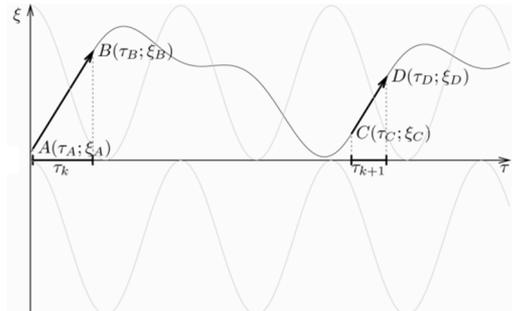


Fig. 3. The qualitative form of the phase trajectories

1.3. Dynamics of the system

For the purpose of modeling the dynamics of system Eqs. (1-2), the function for the

dimensionless belt speed was taken as follows: $\theta(\tau) = A\cos(\Omega\tau) + B$. The function $\varepsilon(\tau_k)$ for the dimensionless coefficient of static friction was taken as follows:

$$\varepsilon(\tau_k) = \begin{cases} \tau_k, & \tau_k \leq \varepsilon_*, \\ \varepsilon_*, & \tau_k \geq \varepsilon_*. \end{cases} \quad (9)$$

Suppose $M_i(\tau_i, \xi_i)$, $i = 0, 1, \dots, n$ – a sequence of point on the surface Π , which does not belong to the strip Π_C and which is determined by the Eq. (5) for $i = 2k < n$, $k = 1, 2, \dots$ and Eq. (6) for $i = 2m + 1 < n$, $m = 0, 1, \dots$. Suppose as well, that the coordinates of M_0 are $\tau = \tau_0$, $\xi = 1 + \varepsilon(\tau_{k,c})$, $\dot{\xi} = \theta(\tau_0)$. For such sequence, a number n can be found, for which $M_{n+1}(\tau_p, \xi_p)$ will belong to the strip Π_C and the interval of phase trajectory is described by Eq. (9) until the point determined from condition Eq. (4). Let's take T_+ to denote the mapping $M_{2k+1} \rightarrow M_{2k+2}$, $k = 0, 1, 2, \dots < n$, and T_- – the mapping $M_{2m} \rightarrow M_{2m+1}$, $m = 1, 2, \dots < n$. The mapping $M_0 \rightarrow M_{n+1}$ can thus be described by $T_1(j, l, n) = ((T_-)^j (T_+)^l) \Big|_{\xi=1}^{\xi_p}$, $l, j = 0, 1, \dots, n$. The connection between two adjacent stick motion intervals τ_k, τ_{k+1} can be described as follows $\psi(\tau_{k+1}) = \varphi(\tau_k)$:

$$\begin{aligned} \psi(\tau) &= -\varepsilon(\tau) + \frac{A}{\Omega} (\sin(\Omega\tau) - \sin(\Omega\tau_p)) - A\Omega\sin(\Omega\tau) + B(\tau - \tau_p), \\ \varphi(\tau) &= 1 + (-1)^{j+1} \xi_p(\tau_0, \tau_1, \dots, \tau_n, \tau), \quad 2(j-1) \leq \varepsilon(\tau) \leq 2, \quad j = 1, 2, \dots \end{aligned}$$

Since the system state trajectory will always end on the sliding surface, they dynamics of the system can be examined using Poincare map of the boundary $\Gamma_1(\Gamma_2)$ into itself, or using the sequence of durations of stick motions τ_k , $k = 1, 2, 3, \dots$. To study the dynamics of the system, we developed an application allowing to perform calculations of trajectories of the system, the Poincare maps, and bifurcation diagrams, using different values of the system parameters. All calculations are done using double precision floating point arithmetic. When calculating a single trajectory, different points on the trajectory are calculated with progressively increasing time distance from the initial point, until the trajectory reaches a boundary between areas where the system behavior is described by different equations, e.g. when the trajectory reaches the surface $\Pi(\dot{\xi} = \theta(\tau))$. Then the accuracy of the calculated point of the transition to a different area is further improved using bisection method, until the desired accuracy is reached. After that, a new trajectory piece is calculated the same way starting from the calculated transition point as the initial point. Trajectory pieces are calculated until the time or the number of stick motion interval reaches the defined limit.

To build a bifurcation diagram, one of the parameters is selected to be varied and its range is selected. Then for each value in the range its own trajectory is calculated, with all the other parameters having the same values for each trajectory. To build the graph, a certain number (usually 30) of last intervals of stick motion are taken and displayed for each trajectory.

1.4. The results of numerical calculations

Fig. 4 displays a bifurcation diagram with Ω as the variable parameter. The other parameters' values are: $\theta(\tau) = 0.1\cos(\Omega\tau) + 1.41$, $\varepsilon_* = 3$. Fig. 5(a) displays a diagram with parameter Ω range from 1 to 5, the other subfigures show different subintervals of that.

Fig. 4 shows that depending on the frequency of the belt movement, the system can display different types of dynamic behavior, including chaos. It should be noted, that with the increase of the frequency chaos become less frequent.

Fig. 5 displays diagrams with A as the variable parameter and $\theta(\tau) = A\cos(\Omega\tau) + 1.41$, $\varepsilon_* = 3$. $\Omega = 1.466$ on Fig. 5(a) and $\Omega = 3.23$ on Fig. 5(b).

Fig. 5 shows the influence of the increase of the amplitude of the belt oscillations on the

dynamics of the system. Two conclusions can be made based on it. One, the increase of the amplitude from zero and within a small vicinity of zero changes the dynamics of the system gradually, without sudden qualitative changes.

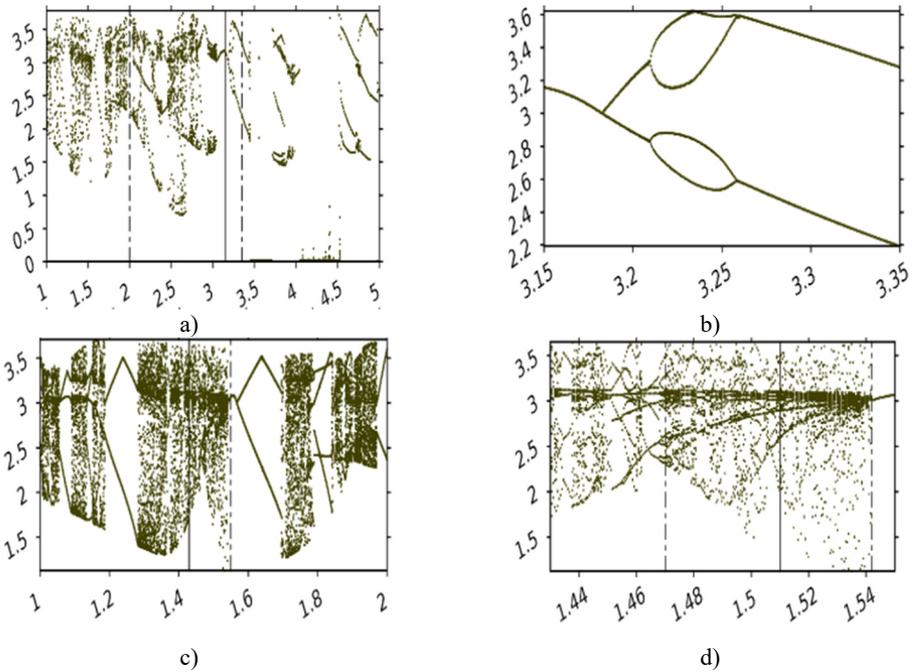


Fig. 4. Bifurcation diagrams in frequency Ω tape oscillation

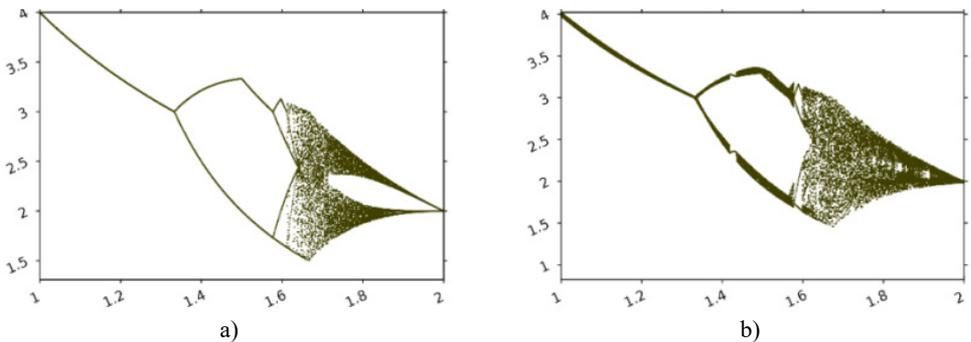


Fig. 5. The bifurcation diagram of the amplitude A fluctuations tape

2. Conclusions

This paper studied the dynamics of a vibration system with hereditary friction. Under the influence of an external periodic force. The obtained bifurcation diagrams of the system revealed the main regimes of periodic and stochastic motions as functions of the governing parameters of the system:

1) As the amplitude of the external force increases, the bifurcation diagrams get “blurred” in relation to the case of no external force. This indicates the appearance of chaos, limited by the amplitude of the external force. As the amplitude of the external force grows further, periodic motions are born with one long stop. In the last case the system behaves as if there was no external force.

2) Increase of the frequency of the external force leads to chaotization of the motions. High

frequencies lead to the birth of periodic motions with a finite number of long stops

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