An optimal placement of PCLD treatment for vibration reduction of plates

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Abstract. An efficient method to reduce the frequency averaged transverse vibration level of a plate by optimizing the position of attached passive constrained layer damping PCLD is presented. This method uses a multilayer anisotropic plate model which is an equivalent single layer plate model (ESL) where transverse shear stresses and displacements are continuous at each layer's interfaces. Hence, for a laminate composed of an arbitrary number of layers, all the quantities are related to those of the first layer. The optimization process is based on the use of the ESL plate model combined with the genetic algorithm (GA) and Latin Hypercube Sampling (LHS) algorithm implemented in a Rayleigh-Ritz resolution program. The convergence acceleration of the method is achieved thanks to the combination of genetic algorithm and the use of the ESL plate model instead of a three-dimensional mesh that requires more computing resource.

Keywords: genetic algorithm, passive constrained layer damping, vibration, plate, Rayleigh-Ritz, equivalent single layer plate model.

1. Introduction

Numerous solutions have been developed in the past to reduce the vibration level of vibrating structures. Two main approaches were usually used: passive methods based on a good knowledge of damping mechanisms to increase energy dissipation and active control methods using electrical devices such as controllers and sensors to reduce the overall level of vibration. Passive treatments, generally, remain appreciated in transportation domain, in particular in aeronautic field where mass reduction is appreciated.

Nowadays, passive constrained layer damping (PCLD) is a classical method for reducing vibrations in vehicles. This is achieved by covering totally or partially vibrating surfaces using patches of viscoelastic materials in order to reduce vibrations and sound radiation arising from local deformation of structures.

Given an existing structure, the optimization of the treatment process of structures by passive patches consists to predict where to add PCLD patches in order to get the most possible reduction in terms of vibration energy of the system, with a limited quantity of added viscoelastic material.

Since the characteristics of the structure change in a major way in the transverse direction, because of the added patches, most methods use Layer-Wise Formulation to describe the vibration behavior of multilayer plates. In this case, displacement fields are expressed for each layer. Hence, during the discretization of the system, the number of degrees of freedom of the system is directly dependent on the number of layers. Although it may be more accurate, this formulation often involves a large number of degrees of freedom, which makes it cumbersome to use in a finite element discretization, especially in an optimization process where the calculation of the objective function is needed multiple times.

The proposed study uses an Equivalent Single Layer (ESL) formulation which describes each component of the displacement field of each layer depending on variables defined in a reference plane. Therefore, displacements of the different layers are described by the x and y coordinates of the reference plane and the normal direction z to the plane x, y.

The optimization process is based on the GA and the ESL model, since the number of added patches do not increase the final number of variables describing vibrational behavior of the overall structure.

Rayleigh-Ritz method was preferred to a classical finite element discretization as it avoids imposing a compulsory compatible mesh with the shape of patches to be placed on the base structure.

2. Multilayer plate model

As mentioned earlier, the ESL formulation allows one to simulate the behavior of a rectangular multi-layered plate with one or several multi-layered patches. It is based on an out-of-plane assumed displacement field obtained by means of kinematic considerations. This approach was used in the early work of Sun and Whitney [1] for multi-layered plates and has been used later for vibroacoustical purposes by Guyader and Lesueur [2].

It is a two-dimensional plate model with the five classical displacement unknowns, but it differs from classical laminate theories (CLT) and the (first-order) shear deformation theory (SDT) because the assumed displacement variation, with respect to z, is piecewise linear. This is the result of writing continuities of both displacements and shear stresses at the interface. For practical reasons, the displacement field of each layer $l \in [2,..., n]$ is linked to the displacement field of the first layer. The displacement field in each layer is written as follows:

$$\begin{cases} u_\alpha^l(x,y,z) = u_\alpha^l(x,y,z^l) + (z^l - z)\left(u_{3,\alpha}^1(x,y) - \gamma_{\alpha 3}^l(x,y)\right), \\ u_3^l(x,y,z) = u_3^1(x,y), \end{cases} \tag{1}$$

where Greek indices stand for in-plane quantities and take values 1 or 2, superscript l stands for the lth layer and superscript 1 stands for the first layer for which all will be related, $\gamma_{\alpha 3}^l$ are the transverse (engineering) shear strains, and z^l is the elevation of the layer l.

Displacement of layer $l+1$ is linked to that of layer l using the continuity at the interfaces, and, recursively, it can be linked to the displacement field of the first layer, following a process detailed in Ref. [2]. As the first layer is common to the uncovered and covered parts of the plate, all the displacements in the multilayered structure are known in terms of the first layers displacement field, including only the five classical plate unknowns: three displacements and two rotations The construction of the discrete motion equation system is achieved by the Rayleigh-Ritz method.

3. Optimization method

3.1. Generic properties of the used method

The optimization method is performed through an iterative process. For a given surface and a given coverage rate α, the optimisation objective is to provide the best arrangement of patches in order to reduce vibration level over a given frequency range using an optimization algorithm:

$$\alpha = \frac{S_P}{S_T}, \tag{2}$$

where S_P is the surface to be covered with patches and S_T is the total surface of the base plate. For each step or generation during the optimization process, simulations are performed using different patch arrangements called samples or populations, each sample i being identified by a set of geometrical parameters to define position and surface of patches over the base plate:

$$S_P = S_1^i + S_2^i + \cdots + S_p^i, \tag{3}$$

where S_j^i being the surface of patch j which is an element of sample i. To evaluate the efficiency of each sample, MSV is averaged over the surface of the plate S_T and integrated over the frequency range ω_2-ω_1:

$$V = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} \int_{S_T} \|\vec{v}(\omega)\|^2 d\omega dS. \tag{4}$$

Each iteration is called a generation. The algorithm tries to propose new samples becoming part of a new generation in order to improve the criteria. The sample issued from this optimization process is taken as the solution of the optimization problem. To obtain this solution, two techniques are combined, Latin Hypercube Sampling (LHS) and GA. The first technique is a random sampling method proposed by McKay et al. [3] as an alternative of simple random sampling in computer experiments, while the second one is a stochastic optimization method based on mechanism of natural evolution and genetic theory. LHS generates an hypercube which is a working space for the involved variables, and proposes samples belonging to this hypercube. The main benefit of the LHS algorithm, compared to random sampling, is that the working space is efficiently sampled with a reduced number of samples. In the present method, the LHS algorithm provides the set of initial samples and also gives new samples which are added to complete the generations at each iteration.

3.2. Used algorithms

The mechanical structure used for this study consists of a plate on which one must place one or more patches to have a maximum reduction of vibration. Plate dimensions are specified in Fig. 1.

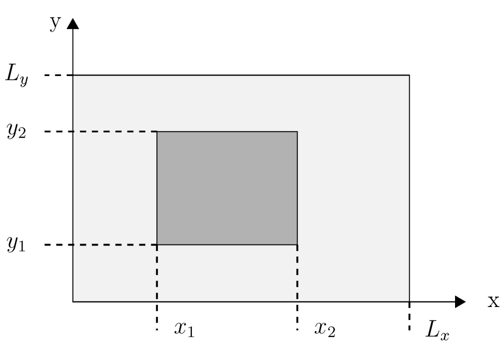

Fig. 1. Patch positioning

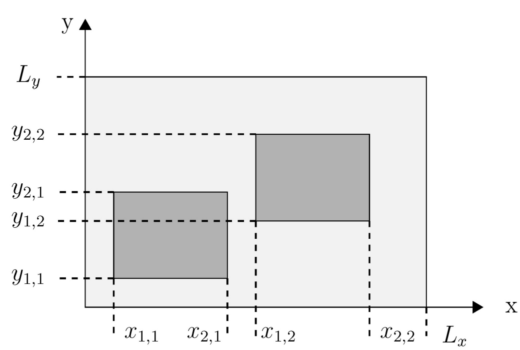

Fig. 2. Patch positioning

3.2.1. Mono-patch optimization

This part concerns positioning of a single patch on the base plate. Surface of the patch to be placed is fixed beforehand, so, each sample consists of four variables called also genes which are the four coordinates x_1, y_1, x_2 and y_2 of the patch relative to the base plate.

To generate the initial population, the LHS method is applied on the variable x_1 which varies between 0 and $L_x/2$ to benefit from the symmetric geometry. Then, for each sample, one seeks the minimum value $x_{2\min}$ of x_2 which corresponds to the position giving a strip with a width x_1-x_2, a maximum height L_y and a surface S_P defined by the prefixed coverage rate $S_P = \alpha \times S_T$ where S_P is the surface of the patch and $S_T = L_x \times L_y$ the surface of the base plate. The final value of x_2 is randomly chosen between the minimum limit $x_{2\min}$ and the maximum limit L_x. To obtain the ordinate y_1 and y_2, the width $L = (y_2 - y_1)$ required for the fixed coverage rate α is used. Then value of the variable y_1 is chosen randomly between 0 and $y_{1\max} = Ly - L$. Finally, y_2 is obtained by the equation: $y_2 = y_1 + L$.

Then, efficiency calculation of samples, their ranking, parent choice, crossover and mutation are obtained in a conventional manner.

Samples to be crossed by a GA crossover operator are chosen by the classical roulette-wheel.

For a couple $((x_1, x_2, y_1, y_2), (x_1', x_2', y_1', y_2'))$, crossover permutations are performed only on genes 2 and 3. To allow the evolution of gene 1, mutation is performed only on x_1. As explained previously, the fourth gene is not a variable, but is deduced from the other parameters. A verification step is performed to ensure that obtained samples by crossover and mutation process satisfy the coverage rate and belonging to the base plate conditions.

3.2.2. Multi-patch optimization

Positioning problem of multiple patches on the base plate, without overlap, introduces several variables, making more complex the optimization. The main difference with the mono-patch problem is the distribution of the covered surface on several patches without overlapping. For a given sample, the total area S_P to be covered is shared randomly on all patches constituting the sample. For this, surfaces S_1 , S_2 , $S_3, \ldots$ have been assigned to different patches 1, 2,… $(S_1 + S_2 + S_3 + \cdots = S_P)$.

Two chromosomes matrices are thus defined. The first matrix contains data relating to surfaces $[S_1, S_2, S_3, \ldots]$ while the second matrix contains position data of each patch. So, sample i is defined as:

$$\left[x_{1,1}^i, x_{2,1}^i, y_{1,1}^i, y_{2,1}^i, x_{1,2}^i, x_{2,2}^i, y_{1,2}^i, y_{2,2}^i, x_{1,3}^i, x_{2,3}^i, y_{1,3}^i, y_{2,3}^i, \ldots\right].$$

The rest of the process is identical to the mono-patch case. For each iteration, one checks that the generated samples consist of non-overlapping patches, and completely included in the base plate. Note that this method becomes complex when the number of patches is important.

3.2.3. Multi-band optimization

According to the previous study concerning the multi-patches optimization, it is noted that the number of variables increases rapidly with the number of patches to be placed, making the optimization method difficult to implement. In this section, considered patches are strips with the same width in order to reduce the number of variables.

Thus the total surface of the base plate is divided into N strips with the same width. A binary vector consisting of 0 or 1 is generated to specify the presence or absence of viscoelastic patch on each strips. The total area of the patches is randomly distributed over all strips which should be covered by patches $S_P = S_1 + S_2 + \cdots + S_N$. In this case, optimization variables are:

- Components of a vector to specify existence or absence of a patch over the strip.
- Distribution of surfaces to be covered on the strips.
- Patch positions over the strips.

Thus, three matrixes are used for optimization variables. The first matrix is used to identify the bands with or without a patch, the second to specify the covered surface of each strip and the last to define the positions of patch over covered strip. The rest of the algorithm is the same as the previous.

4. Numerical simulation

4.1. Tested example

The used structure consists of an aluminum plate, length $L_x = 0.5$ m, width $L_y = 0.4$ m. The four edges of the plate are free. A unit harmonic force is applied on a corner. The excitation frequency range is 10-40 Hz which include the frequency of the first and second bending mode.

Patches to be placed consist of a viscoelastic material and an aluminum constrained plate. Thickness and material characteristics are specified in Table 1.

Table 1. Properties of materials of base beam and PCLD patch

Properties	Base plate	Constraining material	Viscoelastic material
Young modulus, E (GPa)	70	70	–
Loss factor, η (%)	0.1	0.1	–
Density, ρ (kg/m^3)	2700	2700	1000
Poisson ration, ν	0.3	0.3	0.45
Thickness, (mm)	1.5	0.5	0.2

The objective function is the MSV averaged over the surface of the plate and integrated over the excitation frequency band. The coverage rate is 40

The problem was solved with the proposed method using Rayleigh-Ritz instead of the finite element method. Therefore, the form of patches and their numbers are completely independent of the resolution method.

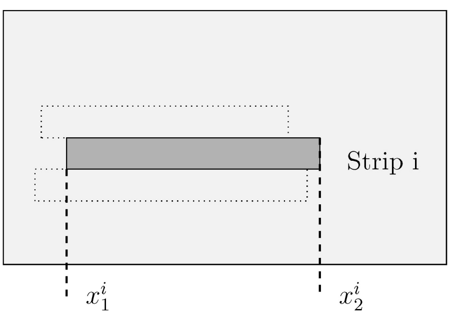

Fig. 3. Patch positioning

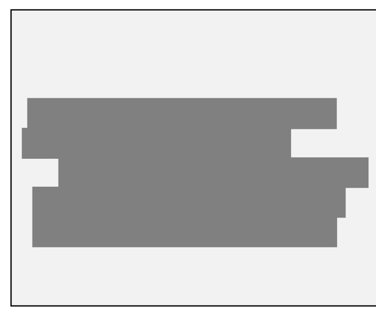

Fig. 4. Multi-band solution

4.2. Results

Quantity, number and shape of patches to be placed on the base plate depend on the chosen optimization algorithm. Mono-patch algorithm works with a single patch, the multi-patch was used with 3 patches and finally the multi-band is applied with a division of 10 strips along the horizontal direction of the plate.

Patches coordinates provided by the optimization process are shown on Table 2.

Table 2. Patches coordinates relative to the base plate

	Patch	x_1 (mm)	x_2 (mm)	y_1 (mm)	y_2 (mm)
Mono-patch	1	40	290	80	400
Multi-patch	1	20	310	190	300
	2	20	320	110	180
	3	20	280	300	400
Multi-band	1	30	430	80	120
	2	30	450	120	160
	3	60	480	160	200
	4	10	370	200	240
	5	20	430	240	280

Fig. 4 is a graphic representation of the arrangement strips supplied from the multi-band optimization algorithm.

Elements of Table 3 show that the algorithms converge toward different solutions but values of the final objective function are roughly identical.

Fig. 5 shows the evolution of the objective function for 1000 iterations performed by three optimization algorithms. It can be seen that the best solution is obtained with Algorithm 2, after it is followed by the algorithm 4 and 3, respectively

Table 3. Reduction rate obtained by the three algorithms

	Function objective (m/s)	Reduction
Mono-patch	1.635	95.94
Multi-patch	1.604	96.02
Multi-band	1.136	97.18

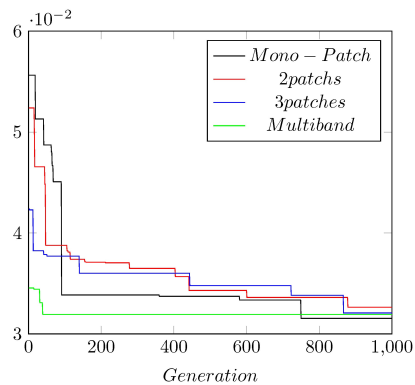

Fig. 5. Objective function decrease according to the number of iterations

5. Conclusions

An optimization method of PCLD patches positions, based on genetic algorithms is presented in this study. This method uses an equivalent single layer (ESL) model, which a well adapted model to optimization problems requiring significant computing time.

Thus a single-patch algorithm that places a single patch to reduce the vibration level in a given frequency bandwidth was developed. This algorithm has been extended to the case of two or more patches. The increased complexity of such algorithms with the number of patches to be placed led us to develop an algorithm that optimizes the position of patches that have the form of strips with a constant width.

The different algorithms have generally converged towards different solutions, but with substantially the same objective functions. All these solutions tends to place PCLD treatment on the structure nodal lines, for a given mode shape, i.e. on the areas of high strain energy.

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