2. Mathematically study on vibration of visco-elastic parallelogram plate

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Abstract. The free vibration of parallelogram plate with varying thickness and thermal effect are investigated in the present study. Using the separation of variables method, the governing differential equation has been solved for vibration of visco-elastic orthotropic parallelogram plate. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two-term deflection function. The frequencies corresponding to the first two modes of vibrations are obtained for a parallelogram plate for different values of taper constant and thermal gradient.

Keywords: visco-elastic, vibration, parallelogram plate, frequency, taper constant.

1. Introduction

Visco-elastic behaviour may be defined as material response that exhibits characteristic of both a viscous fluid and an elastic solid. A visco-elastic material combines these two properties it returns to its original shape after being stressed, but does it slowly enough to oppose the next cycle of vibration. The degree to which a material behaves either viscously or elastically depends mainly on temperature and rate of loading (frequency). Many polymeric materials (plastic, rubbers, acryllics, silicones, vinyls, adhesive, etc.) having long chain molecules exhibit visco-elastic behaviour. The dynamic properties (shear modulus, extensional modulus, etc.) of linear visco-elastic materials can be represented by the complex modulus approach.

Plates of various geometries are commonly used as structural elements in various fields of engineering such as civil, naval and mechanical. Their design requires an accurate determination of their natural frequencies and mode shapes. In particular, rectangular plates are widely used in ocean structures and aerospace industry. Plates with varying thickness possess a number of attractive features such as material saving, weight reduction, stiffness enhancing, high strength and also meet the desirability of economy.

Vibration effects have always been a principle concern of engineers. In the epoch of science and technologies it is desired to design large machines with smooth operation and unwanted vibrations. Sometimes unwanted vibration causes fatigues. Unwanted vibration can damage electronic components of aerospace system, damage buildings by earthquake, bring tsunami, and contribute to toppling of tall smokestacks, collapse of a suspension bridge in a windstorm. There are a multitude of applications where vibration effect is required e.g. in string and percussion instruments, in the design of loudspeakers, space shuttles, satellites where discrepancies in the temperature also affects the vibration effect. Controlled vibration effects are also required in health industry, paper industry, design of structures, building construction, reducing soil adhesion and many more areas engross vibration upshot.


Hence vibrations totally affect our day-to-day life. Thus for design engineers and scientist, it has always been a necessity to optimize or to control the effect of unwanted vibrations as much as possible. Present work is a full-fleshed endeavour to assist the design officers, industry people to come up to the situation.

The object of the present study is to determine the effect of a constant thermal gradient on the frequencies of a clamped parallelogram plate with linearly varying thickness. All the edges are taken as clamped. The Rayleigh-Ritz technique has been used to determine the frequencies equation of the plate. The frequency to the first two modes of vibration is obtained for a clamped parallelogram plate for various values of thermal gradient and taper constant.

2. Parallelogram plate and analysis

The parallelogram plate $Z$ is shown in the Fig. 1 with oblique and rectangular co-ordinate system.

The parallelogram plate $Z$ be defined by the three number $a$, $b$ and $\theta$ as shown Fig. 1. The special case of rectangular plate follows by putting $\theta = 0^\circ$, here:

$$\xi = x - y\tan \theta, \quad \eta = y\sec.$$

(1)
Bending and twisting moments of visco-elastic parallelogram plate of variable thickness are related to displacement by:

\[
\begin{align*}
M_\xi &= -\ddot{D}[D\xi w_{\xi\xi\xi} + D_1 \sec2\theta (\sin2\theta w_{\xi\xi} - 2\sin\theta w_{\xi\eta} + w_{\eta\eta})], \\
M_\eta &= -\ddot{D}[D\eta w_{\eta\eta\eta} + D_\eta \sec2\theta (\sin2\theta w_{\eta\eta} - 2\sin\theta w_{\xi\eta} + w_{\xi\xi})], \\
M_{\xi\eta} &= -2\ddot{D}\xi\eta \sec2\theta [w_{\xi\eta\eta} - \sin\theta w_{\xi\xi}],
\end{align*}
\]

(2)

where, \(D\xi\) and \(D\eta\) are flexural rigidities of plate, \(D\xi\eta\) is torsional rigidity of plate, \(\ddot{D}\) is unique rheological operator.

The governing differential equation of transverse motion of visco-elastic orthotropic parallelogram plate of variable thickness, \(\xi\)- and \(\eta\)- co-ordinates, is [7]:

\[
\ddot{D}[(D_\xi + D_\eta\tan4\theta + 2H\tan2\theta)w_{\xi\xi\xi\xi} - 4(\sec\theta)\eta\tan3\theta + H\tan\theta)w_{\xi\xi\eta\eta} + (sec\theta)(6D_\eta\tan2\theta + 2H)w_{\xi\xi\eta\eta} + 4(D_\eta\tan\theta\sec3\theta)w_{\xi\eta\eta\eta} + (D_\xi\sec4\theta)w_{\eta\eta\eta\eta} + 2(H_\xi\tan2\theta - H_\eta\tan\theta + D_\xi\xi - D_\eta\eta\tan3\theta)w_{\xi\eta}\xi\xi \\
-2(\sec\theta)(2H_\xi\tan\theta - H_\eta\tan\theta - 3D_\eta\eta\tan2\theta)w_{\xi\eta}\xi\eta \\
+2\sec2\theta(H_\xi\xi\tan\theta - 3D_\eta\eta\tan\theta)w_{\xi\eta}\eta + 2(D_\eta\eta\sec3\theta)w_{\xi\eta}\eta \\
+(D_\xi\xi + D_\eta\xi\xi\tan4\theta - 2D_\eta\xi\xi\sec\theta\tan3\theta + D_\eta\eta\eta\sec2\theta\tan2\theta \\
+ 3D_\eta\xi\xi\tan2\theta - 2D_\eta\xi\xi\sec\theta\tan3\theta + D_\xi\eta\sec\theta\tan2\theta \\
- 4D_\eta\eta\xi\xi\sec3\theta\tan3\theta + 4D_\eta\eta\eta\sec2\theta\tan2\theta \\
- 2D_\eta\eta\eta\sec3\theta\tan2\theta - 2D_\xi\xi\xi\sec\theta\tan3\theta - 4D_\xi\xi\xi\sec2\theta\tan2\theta \\
+ 4D_\eta\eta\xi\xi\sec2\theta\tan3\theta + (D_\eta\eta\xi\xi\sec2\theta\tan2\theta - 2D_\eta\eta\eta\sec3\theta\tan\theta \\
+ D_\eta\eta\eta\sec4\theta + D_\xi\xi\xi\sec2\theta)w_{\eta\eta\eta} + \rho w_{\xi\eta\xi\eta} = 0.
\]

A comma followed by a suffix denotes partial differential with respect to that variable. The solution of Eq. (3) can be taken in the form of products of two functions as for free transverse vibration of the visco-elastic orthotropic parallelogram plate, \(w(\xi, \eta, t)\) can be expressed as:

\[
w(\xi, \eta, t) = W(\xi, \eta)T(t),
\]

(4)

where \(T(t)\) is the time function and \(W\) is the maximum displacement with respect to time \(t\). Substituting Eq. (4) into Eq. (3), one obtains:

\[
\ddot{D}[(D_\xi + D_\eta\tan4\theta + 2H\tan2\theta)W_{\xi\xi\xi\xi\eta\eta} - 4(\sec\theta)\eta\tan3\theta + H\tan\theta)W_{\xi\xi\eta\eta\eta\eta} + (sec\theta)(6D_\eta\tan2\theta + 2H)W_{\xi\xi\eta\eta\eta\eta} + 4(D_\eta\tan\theta\sec3\theta)W_{\xi\eta\eta\eta\eta\eta} + (D_\xi\sec4\theta)W_{\eta\eta\eta\eta\eta\eta} + 2(H_\xi\tan2\theta - H_\eta\tan\theta + D_\xi\xi - D_\eta\eta\tan3\theta)W_{\xi\eta}\xi\xi \\
-2(\sec\theta)(2H_\xi\tan\theta - H_\eta\tan\theta - 3D_\eta\eta\tan2\theta)W_{\xi\eta}\xi\eta \\
+2\sec2\theta(H_\xi\xi\tan\theta - 3D_\eta\eta\tan\theta)W_{\xi\eta}\eta + 2(D_\eta\eta\sec3\theta)W_{\xi\eta}\eta \\
+(D_\xi\xi + D_\eta\xi\xi\tan4\theta - 2D_\eta\xi\xi\sec\theta\tan3\theta + D_\eta\eta\eta\sec2\theta\tan2\theta \\
+ 3D_\eta\xi\xi\tan2\theta - 2D_\eta\xi\xi\sec\theta\tan3\theta + D_\xi\eta\sec\theta\tan2\theta \\
- 4D_\eta\eta\xi\xi\sec3\theta\tan3\theta + 4D_\eta\eta\eta\sec2\theta\tan2\theta \\
- 2D_\eta\eta\eta\sec3\theta\tan2\theta - 2D_\xi\xi\xi\sec\theta\tan3\theta - 4D_\xi\xi\xi\sec2\theta\tan2\theta \\
+ 4D_\eta\eta\xi\xi\sec2\theta\tan3\theta + (D_\eta\eta\xi\xi\sec2\theta\tan2\theta - 2D_\eta\eta\eta\sec3\theta\tan\theta \\
+ D_\eta\eta\eta\sec4\theta + D_\xi\xi\xi\sec2\theta)W_{\eta\eta\eta\eta} + \rho \dot{w}_{\xi\eta\xi\eta} = 0.
\]

(5)

The proceeding equation is satisfied if both of its sides are equal to a constant. Denoting this constant by \(p^2\), we get two equations:
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\[ \bar{D} \left[ (D_{\xi} + D_{\eta} \tan 4\theta + 2H\tan 2\theta)W_{\xi\xi\eta\eta} - 4(\sec \theta)(D_{\eta}\tan 3\theta + H\tan \theta)W_{\xi\xi\eta\eta} \\
+ (\sec \theta)(6D_{\eta}\tan 2\theta + 2H)W_{\xi\eta\eta\eta} - 4(D_{\eta}\tan \theta\sec 3\theta)W_{\xi\eta\eta\eta} \\
+ (D_{\eta}\sec 4\theta)W_{\eta\eta\eta\eta} + 2(H_{\xi} \tan 2\theta - H_{\eta} \tan \theta + D_{\xi}\xi - D_{\eta}\eta \tan 3\theta)W_{\xi\xi\eta\eta} \\
- 2(\sec \theta)(2H_{\xi} \tan \theta - H_{\eta} \tan \theta - 3D_{\eta} \tan 2\theta)W_{\xi\xi\eta\eta} \\
+ 2\sec 2\theta(H_{\xi} - 3D_{\eta} \tan \theta)W_{\xi\eta\eta\eta} + 2(D_{\eta} \sec 3\theta)W_{\eta\eta\eta\eta} \\
+ (D_{\xi,\xi} + D_{\eta,\xi} \tan 4\theta - 2D_{\eta,\xi} \sec \theta \tan 3\theta + D_{\eta,\eta} \sec 2\theta \tan 2\theta \\
+ 3D_{\xi,\eta} \tan 2\theta - 2D_{\xi,\xi} \sec \theta \tan \theta + D_{\eta,\eta} \sec \theta + 4D_{\xi,\xi} \xi \tan 2\theta \\
- 4D_{\xi,\eta} \eta \sec \theta \tan \theta)W_{\xi\xi\eta\eta} - 2D_{\eta,\eta} \xi \xi \xi \sec \theta \tan 3\theta + 4D_{\eta,\eta} \eta \xi \sec 2\theta \tan 2\theta \\
- 2D_{\eta,\eta} \xi \eta \xi \eta \sec \theta \tan \theta - 2D_{\eta,\xi} \xi \eta \xi \eta \sec \theta \tan 2\theta + 4\xi \sec 2\theta)W_{\xi\eta\eta\eta} \\
+ (D_{\eta,\xi} \sec 2\theta \tan 2\theta - D_{\eta,\xi} \eta \sec 3\theta \tan \theta + D_{\eta,\eta} \sec 4\theta + D_{\eta,\xi} \xi \sec 2\theta)W_{\eta\eta\eta\eta} \right] \\
- \rho 2hW = 0. \tag{6} \]

Eqs. (6) is the differential equation of motion for visco-elastic orthotropic parallelogram plate of variable thickness.

2.1. Equation of motion

The expressions for the strain energy, \( S_{\text{max}} \), and kinetic energy, \( K_{\text{max}} \), in the visco-elastic parallelogram plate when executing transverse vibration of mode shape \( W(\xi, \eta) \) are [7]:

\[
S_{\text{max}} = \left( \frac{1}{2} \right) \int_{a}^{b} \int_{0}^{b} \left[ D_{\xi}(W_{\xi\xi}) + D_{\eta}(W_{\eta\eta}) \tan 2\theta - 2W_{\xi\eta} \tan \theta \sec \theta + W_{\eta\eta} \sec 2\theta \right] \frac{a}{2} \frac{b}{2} \\
+ \frac{2a}{2} \frac{b}{2} \int_{0}^{b} \int_{0}^{b} \left( W_{\xi\xi} \tan 2\theta + 2W_{\xi\eta} \tan \sec \theta + W_{\eta\eta} \sec 2\theta \right) + 4D_{\xi}(\xi \xi \xi \sec \theta) - W_{\xi\xi} \tan \theta \\
+ W_{\eta\eta} \sec \theta) \cos \theta d\eta d\xi, \tag{7} \]

\[
K_{\text{max}} = \left( \frac{1}{2} \right) \rho p^{2} \int_{0}^{b} \int_{0}^{b} \left( hW \right) \cos \theta d\eta d\xi. \tag{8} \]

Assuming that parallelogramic plate of engineering material has a steady two dimensional linear temperature distribution i.e.:

\[
\tau = \tau_{0} \left( 1 - \frac{\xi}{a} \right) \left( 1 - \frac{\eta}{b} \right), \tag{9} \]

where \( \tau \) denotes the temperature excess above the reference temperature at any point on the plate and \( \tau \) denotes the temperature at any point on the boundary of plate and \( a \) and \( b \) are the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in the:

\[
E = E_{0}(1 - \gamma \tau), \tag{10} \]

where, \( E_{0} \) is the value of the Young’s modulus at reference temperature i.e. \( \tau = 0 \) and \( \gamma \) is the slope of the variation of \( E \) with \( \tau \). The modulus variation become:

\[
E = E_{0} \left[ 1 - \alpha \left( 1 - \frac{\xi}{a} \right) \left( 1 - \frac{\eta}{b} \right) \right], \tag{11} \]

where \( \alpha = \gamma \tau_{0} \) (0 ≤ \( \alpha \) ≤ 1), thermal gradient.

Now, thickness variation of visco – elastic orthotropic parallelogram plate parabolically in \( \xi \) and \( \eta \) directions, as:
\[ h = h_0 \left[ 1 + \beta_1 \left( \frac{\xi}{a} \right) \right] \left[ 1 + \beta_2 \left( \frac{\eta}{b} \right) \right], \]  \hspace{1cm} (12)

where \( \beta_1 \) and \( \beta_2 \) is the taper constant in \( \xi \) and \( \eta \) directions and \( h_0 = h \) at \( \xi = \eta = 0 \).

### 2.2. Equation of frequency and its solution

Rayleigh-Ritz technique requires that maximum strain energy be equal to the maximum kinetic energy. So it is necessary for the problem consideration that:

\[ \delta(S_{\text{max}} - K_{\text{max}}) = 0, \]  \hspace{1cm} (13)

for arbitrary variations of \( W \) are satisfying relevant geometrical boundary conditions.

For a visco-elastic orthotropic parallelogram plate clamped (C-C-C-C) along all the four edges, the boundary conditions are:

\[ W = W_{\xi} = 0 \text{ at } \xi = 0, a \text{ and } \eta = 0, b, \]  \hspace{1cm} (14)

and the corresponding two-term deflection function is taken as:

\[ W = \left[ \left( \frac{\xi}{a} \right) \left( \frac{\eta}{b} \right) \left( 1 - \frac{\xi}{a} \right) \left( 1 - \frac{\eta}{b} \right) \right] 2 \left[ A_1 + A_2 \left( \frac{\xi}{a} \right) \left( \frac{\eta}{b} \right) \left( 1 - \frac{\xi}{a} \right) \left( 1 - \frac{\eta}{b} \right) \right]. \]  \hspace{1cm} (15)

Now assuming the non-dimensional variable as:

\[ X = \frac{\xi}{a}, \quad Y = \frac{\eta}{a}, \quad h = \frac{h}{a}, \quad W = \frac{W}{a}, \]  \hspace{1cm} (16)

and:

\[ E_1^* = \frac{E_1}{1 - \nu_\xi \nu_\eta}, \quad E_2^* = \frac{E_2}{1 - \nu_\xi \nu_\eta}, \quad E^* = \nu_\xi E_2^* = \nu_\eta E_1^*, \]  \hspace{1cm} (17)

and component of \( E_1^* \), \( E_2^* \), \( E^* \) and \( G \) are \( E_1^*_1 \), \( E_2^*_2 \sec \theta \), \( E^*_2 \sec \theta \) and \( G \sec \theta \) respectively \( \xi \)- and \( \eta \)-direction. Using Eqs. (12), (15), (16) and (17) in Eqs. (7) and (8), then substituting the values of \( K_{\text{max}} \) and \( S_{\text{max}} \) from Eqs. (7) and (8) in Eq. (13), one obtains:

\[ \partial(S_1 - \lambda^2 K_1) = 0, \]  \hspace{1cm} (18)

where:

\[ S_1 = (1 + \beta_1 X)^3 (1 + \beta_2 Y)^3 \left[ (\cos^4 \theta + (E^*_2/E^*_1)^2 \sin^4 \theta + 2(E^*/E^*_1)^2 \sin^2 \theta \cos^2 \theta \right. \right. \right. \right. \right. \right. \right.
\[ + 4(G_0/E^*_1)^2 \sin^2 \theta \cos^2 \theta \right] W^2_{XX} \int_0^{b/a} \int_0^h 2 \left[ (E^*_2/E^*_1)^2 \sin^2 \theta + (E^*/E^*_1)^2 \cos^2 \theta \right] W_{XX} W_{YY} dY dX, \]  \hspace{1cm} (19)

\[ -4 \left\{ \left( \frac{E^*_2}{E^*_1} \right)^2 \sin^3 \theta + 2 \left( \frac{E^*}{E^*_1} \right)^2 \sin \theta \cos^2 \theta \right\} \right] W_{XX} W_{YY} \]  \hspace{1cm} (19)

\[ -4 \left[ \frac{E^*_2}{E^*_1} \right]^2 \sin \theta W_{YY} W_{XX} \int_0^{b/a} dY dX, \]  \hspace{1cm} (19)

\[ K_1 = \int_0^h \int_0^h [(1 + \beta_1 X)(1 + \beta_2 Y)W] dY dX, \]  \hspace{1cm} (20)
and \( \lambda^2 = \left( E_1 h_0^2/12a^2 \rho c^2 \right) \).

Here limit of \( X \) and \( Y \) is 0 to 1 and 0 to \( b/a \) respectively.

But Eq. (17) involves the unknown \( A_1 \) and \( A_2 \) arising due to the substitution of \( W(\xi, \eta) \) from Eq. (14). These two constants are to be determined from Eq. (18), as follows:

\[
\frac{\partial (V_1 - \lambda^2 T_1)}{\partial A_n} = 0, \quad n = 1, 2, \ldots
\]  
(21)

Eq. (21) simplifies to the form:

\[
b_{n1} A_1 + b_{n2} A_2 = 0, \quad n = 1, 2,
\]  
(22)

where \( b_{n1}, b_{n2} (n = 1, 2) \) involve parametric constants and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of Eq. (22) must be zero. So gets the frequency equation as:

\[
\begin{vmatrix}
   b_{11} & b_{12} \\
   b_{21} & b_{22}
\end{vmatrix} = 0.
\]  
(23)

From Eq. (23), one can obtain quadratic equation in \( \lambda^2 \) can be found.

3. Result and discussion

The frequency Eq. (23) is a quadratic equation in \( \lambda^2 \) from which values of \( \lambda^2 \) can be found. The frequency parameter \( \lambda \) corresponding to the first two modes of vibration of a clamped parallelogram plate has been computed for various values aspect ratio \( (a/b) \), thermal constant \( (\alpha) \), taper constant \( (\beta_1 \text{ and } \beta_2) \) and skew angle. The Poisson’s ratio \( (\nu) \) is taken as 0.3. These results are summarized in Tables 1-4.

Tables 1 contains the value of frequency parameter of a clamped parallelogram plate for different values of thermal constant \( (\alpha) \) for fixed value of aspect ratio \( (a/b) \) = 1.0 and taper constants \( \beta_1 = \beta_2 = 0.4 \) for the first two modes of vibration for two values of skew angle \( (\theta) \).

TABLE 1. Frequency parameter \((\lambda)\) of a clamped parallelogram plate
for different value of thermal constant \((\alpha)\)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(a/b = 1.0, \theta = 0^\circ) and (\beta_1 = \beta_2 = 0.4)</th>
<th>(a/b = 1.0, \theta = 60^\circ) and (\beta_1 = \beta_2 = 0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>Second mode</td>
<td>First mode</td>
</tr>
<tr>
<td>0.0</td>
<td>40.58</td>
<td>160.98</td>
</tr>
<tr>
<td>0.2</td>
<td>38.68</td>
<td>154.26</td>
</tr>
<tr>
<td>0.4</td>
<td>36.92</td>
<td>146.86</td>
</tr>
<tr>
<td>0.6</td>
<td>34.89</td>
<td>139.87</td>
</tr>
<tr>
<td>0.8</td>
<td>32.78</td>
<td>131.75</td>
</tr>
<tr>
<td>1.0</td>
<td>30.62</td>
<td>123.66</td>
</tr>
</tbody>
</table>

Table 2 gives the value of frequency parameter of a clamped parallelogram plate for different values of aspect ratio \( (a/b) \) and the combinations of:

- \( \alpha = 0.0, \beta_1 = \beta_2 = 0.0 \) and \( \theta = 0^\circ \).
- \( \alpha = 0.0, \beta_1 = \beta_2 = 0.4 \) and \( \theta = 60^\circ \).

It is seen from the tables that as aspect ratio increases frequency parameter increases in all the cases for both modes of vibration.

Table 3 have the value of frequency parameter of a clamped parallelogram plate for different values of taper constant \( (\beta_1) \) for fixed value of \( a/b = 1.0, \theta = 0^\circ, \alpha = 0.4 \) and \( \beta_2 = 0.4 \). It is seen from the tables that as taper constant increases frequency parameter increase in all the cases for both modes of vibration.
Table 4 contains the value of frequency parameter of a clamped parallelogram plate for different values of skew angle ($\theta$) and the combinations of:

- $a/b = 1.0, \alpha = 0.0$ and $\beta_1 = \beta_2 = 0.0$.
- $a/b = 1.0, \alpha = 0.4$ and $\beta_1 = \beta_2 = 0.4$.

It is seen from the tables that as skew angle increases frequency parameter increases in all the cases for both modes of vibration.

### Table 2. Frequency parameter ($\lambda$) of a clamped parallelogram plate for different value of aspect ratio ($a/b$)

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$\alpha = 0.0, \theta = 0^\circ$ and $\beta_1 = \beta_2 = 0.2$</th>
<th>$\alpha = 0.0, \theta = 60^\circ$ and $\beta_1 = \beta_2 = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First mode</td>
<td>Second mode</td>
</tr>
<tr>
<td>0.5</td>
<td>117.14</td>
<td>26.52</td>
</tr>
<tr>
<td>1.0</td>
<td>156.52</td>
<td>39.22</td>
</tr>
<tr>
<td>1.5</td>
<td>267.82</td>
<td>67.08</td>
</tr>
<tr>
<td>2.0</td>
<td>439.77</td>
<td>109.12</td>
</tr>
<tr>
<td>2.5</td>
<td>666.12</td>
<td>163.88</td>
</tr>
<tr>
<td>3.0</td>
<td>892.33</td>
<td>211.04</td>
</tr>
</tbody>
</table>

### Table 3. Frequency parameter ($\lambda$) of a clamped parallelogram plate for different value of taper constant ($\beta_1$)

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>$a/b = 1.0, \theta = 0^\circ, \alpha = 0.4$ and $\beta_2 = 0.4$</th>
<th>$a/b = 1.0, \theta = 0^\circ, \alpha = 0.4$ and $\beta_2 = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First mode</td>
<td>Second mode</td>
</tr>
<tr>
<td>0.0</td>
<td>31.04</td>
<td>124.88</td>
</tr>
<tr>
<td>0.2</td>
<td>32.88</td>
<td>135.03</td>
</tr>
<tr>
<td>0.4</td>
<td>36.12</td>
<td>146.11</td>
</tr>
<tr>
<td>0.6</td>
<td>40.32</td>
<td>158.88</td>
</tr>
<tr>
<td>0.8</td>
<td>42.88</td>
<td>172.08</td>
</tr>
<tr>
<td>1.0</td>
<td>46.23</td>
<td>176.13</td>
</tr>
</tbody>
</table>

### Table 4. Frequency parameter ($\lambda$) of a clamped parallelogram plate for different value of Skew angle ($\theta$)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$a/b = 1.0, \alpha = 0.0$ and $\beta_1 = \beta_2 = 0.0$</th>
<th>$a/b = 1.0, \alpha = 0.4$ and $\beta_1 = \beta_2 = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First mode</td>
<td>Second mode</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>139.01</td>
<td>34.01</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>192.93</td>
<td>48.99</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>301.01</td>
<td>78.11</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>623.98</td>
<td>163.88</td>
</tr>
</tbody>
</table>

### 4. Conclusion

The Rayleigh-Ritz technique has been applied to study the effect of the taper constants on the vibration of clamped visco-elastic isotropic parallelogram plate with linearly varying thickness in two directions on the basis of classical plate theory.

On comparison with [7], it is concluded that the value of the frequency comes smaller in this paper when we compare both the paper results.

In this way, authors concluded that linear variation in both directions is more useful than parabolic variation in one direction.

### References

2. MATHEMATICALLY STUDY ON VIBRATION OF VISCO-ELASTIC PARALLELOGRAM PLATE.
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