

# 1171. Local Hölder exponent analysis of aeroengine dynamics

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**Abstract.** This paper describes the application of the local Hölder exponent which can measure sensitive features associated with nonstationary and nonlinearity. By investigating in detail the structure and the main properties of the local Hölder function, a fault diagnosis technology is developed on the basis of error function. The study is conducted for both engines of two Boeing 737 commercial aircrafts as a measure the regularity of aeroengine time series. In order to accurately detect the impending aeroengine faults, the Hölder exponent estimation is performed for comparative analysis of the aeroengine records. Using this analysis, the difference between left and right engine is obvious when one of the engines is fracture failure.

**Keywords:** Hölder exponent, engine performance, error function, comparative analysis.

## 1. Introduction

In recent years, the fractal and multifractal theory has been studied widely to measure the regularity of time series. The local Hölder exponent, as an important tool in multifractal analysis, provides information about the regularity of a signal [1-4]. But other characterizations of local regularity exist, such as the pointwise Hölder exponent, the oscillation exponents, the fluctuation exponents, Hausdorff dimensions and the degree of fractional differentiability [5, 6]. We shall mainly be concerned in this paper with the study of the local Hölder exponent.

The pointwise Hölder exponent can pertain to the global regularity of a signal, which can be found through the use of the Fourier transform [7]. Unfortunately, the Fourier transform approach cannot be used to find the regularity at a particular point in time. On the other hand, Wavelets are well localized in time and can therefore provide an estimate of the Holder regularity both over time intervals and at specific time points. The formalism, by applying the wavelet transform representation of a signal to multifractal analysis, developed by Arneodo et al in the early nineties [8, 9] has been extensively used to test many natural phenomena and has contributed to substantial progress in each domain in which it has been applied [10-12].

The Hölder exponent has proven to be a useful tool for time series analysis [10, 13] and also have been used in some applications of health monitoring. By fitting an autoregressive moving average (ARMA) model to the wavelet-transformed data, analysis of the Hölder exponent based on wavelet transform is used to indicate the presence of fatigue cracks in gears [3]. By using the wavelet modulus maxima method, the Hölder exponent is used to examine shaft orbits [4].

In this paper, we would notice the Hölder regularity, quantifies the amount of regularity any given point on a signal, using known Hölder exponent [10, 13-15]. Nevertheless the pointwise Hölder exponent provides only global estimates of scaling, while there are cases when local information about scaling provides more relevant information than the global spectrum. Therefore, we address the problem of estimation of the local scaling exponent. Here we first investigate the fractal scaling characteristic of aeroengine dynamics from normal engine by applying the local Hölder exponent method. And then, we obtain the fractal scaling properties of signals from the fracture failure in engine, and compare the difference in the fractal scaling results. To this end, we define error function between the Hölder exponent of original data for left and right engine. The results indicate that the fracture failure engine data can be discovered.

The structure of the paper is as follows. In section 2, we focus on the flight data and the Hölder

exponent analysis. In section 3, we introduce a technique named fractal comparative diagnostics analysis method enabling us to estimate the parameter characteristic for aeroengine. In particular, the ability to distinguish between normal and abnormal signals in engine is demonstrated by two aircraft. The conclusion of the paper is drawn in section 4. Based on the fractal theory and comparative analysis, this paper presents applications of the proposed technique to the aeroengine data.

## 2. Methodology and algorithm

### 2.1. EGT data acquisition and signal processing

Data available from commercial 737 aircrafts are used for this paper. The 737 aircraft data includes operational flight data from on-board flight data recorders. These recorders are part of the Aircraft Condition Monitoring System (ACMS) such as Smart ACMS Recorder (SAR) and Quick Access Recorder (QAR).

SAR data records specific flight parameters on preset data recording channels. But the QAR data is a more comprehensive that data includes an extensive list of flight parameters recorded at specific sampling intervals which are set by the manufacturer. Moreover, the set of data includes engine reports which are generated when selected parameters meet certain predefined conditions during a particular flight phase. Furthermore, the data also includes baseline values for a subset of flight parameters and the respective threshold operating values for these parameters. In addition the series information can come in handy to decide at what stage during a flight, the parameters corresponding to a system or sub-system reach abnormal operating conditions. Therefore, the QAR data is applied in this paper.

By applying flight condition data and situational parameters, diagnostic and prognostic schemes can be developed which contribute to determining engine behavior during a specific flight and predict engine performance by estimating the flight parameter conditions of future flights. Various flight parameters such as Exhaust Gas Temperature (EGT), Fuel Flow (FF), Engine Fan Speeds (N1 and N2) are usually used to estimate the engine health and performance [16].

### 2.2. Analytical method

The aeroengine dynamics have to be considered as nonstationary. Therefore, most of the widely used signal processing techniques based on the assumption of stationarity and globally characterise signals are not fully suitable for detecting short-duration dynamic phenomena [17]. Thus, we address the problem of estimation of the local scaling exponent.

For convenience, we describe briefly the Hölder exponent firstly. Given a function  $f(t)$  with a singularity at time  $t_0$ , the Hölder exponent  $h(t_0)$  defined as the supremum exponent such that there exists polynomial  $P_n(t - t_0)$  of order  $n < h(t_0)$  satisfying [8-11]:

$$|f(t) - P_n(t - t_0)| \leq C|t - t_0|^{h(t_0)}, \quad (1)$$

where  $t$  in the neighborhood of  $t_0$ , and  $C$  is a constant.  $P_n(t)$  is often associated with the Taylor expansion of  $f(t)$  around  $t_0$ , but Eq. (1) is valid even if such expansion does not exist [18]. The Hölder exponent is therefore a function defined for each point of  $f$ , and it describes the local regularity of the function  $f$ . By selecting properly a wavelet kernel  $\psi(t)$  that is orthogonal to polynomials up to degree  $n$ , the continuous wavelet transform (WT) of  $f(t)$  at  $t = t_0$  is expressed as:

$$WT[f](t_0, a) = \frac{1}{a} \int f(t) \overline{\psi\left(\frac{t - t_0}{a}\right)} dt. \quad (2)$$

Further,

$$WT[f](t_0, a) \propto |a|^{h(t_0)}, \quad a \rightarrow 0. \quad (3)$$

The local Hölder exponent  $h(t_0)$  of strength of singularity can be evaluated as the scaling power of the wavelet coefficient at  $t = t_0$  for  $a \rightarrow 0$  [19].

### 3. Engine health analysis

The effectiveness of the proposed technique in perceiving the fracture failure will be discussed in this section. The exhaust gas temperature (EGT), offered by a Commercial Company in Beijing, are applied. Specifically, we consider time series of two plane, aircraft A and aircraft B, in this paper. Fig. 1 shows a section of the normal EGT data from aircraft A.

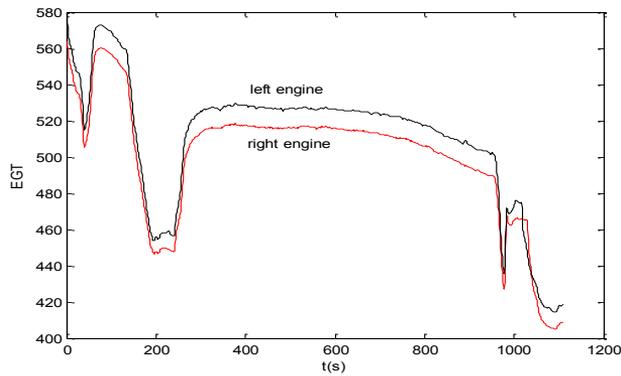


Fig. 1. The EGT data for normal engine of left and right in aircraft A

#### 3.1. Example 1: Aircraft A

In order to test the robustness of the proposed fractal comparative diagnostics method, cruise mode time series of aircraft A are used in this section.

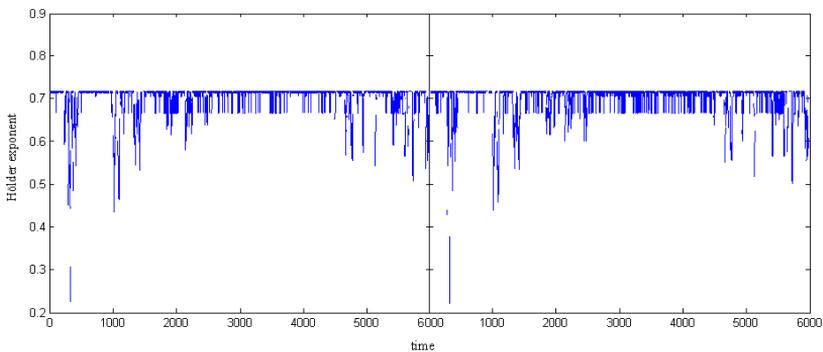


Fig. 2. The Hölder exponents of (a) left engine and (b) right engine

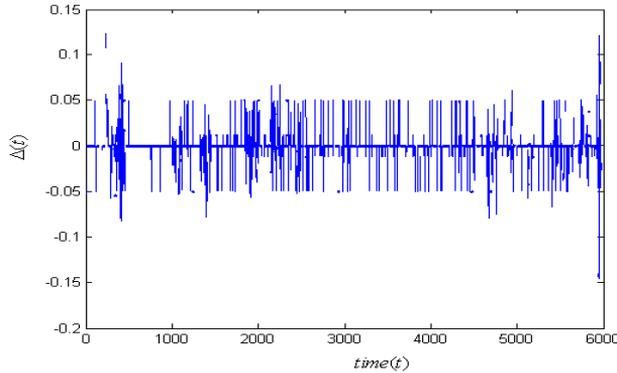
Here, we apply the Hölder exponent analysis method to signal of left and right engine for aircraft A, see Fig. 2, where one of the engine is fracture failure. We observe that there are different scaling phenomena in the plot. And then, we can study the difference between the Hölder exponents of left and right engine by using error function. The error function is defined as:

$$\Delta(t) = |h_{left}(t) - h_{right}(t)|, \quad (4)$$

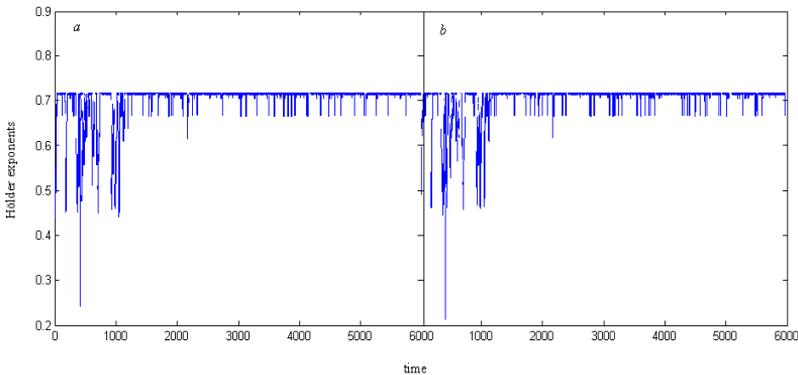
where  $h_{left}(t)$  are these Hölder exponents of left engine,  $h_{right}(t)$  are Hölder exponents of right engine respectively.

Fig. 3 shows the nontrivial difference  $\Delta_f(t) = |h_{left}(t) - h_{right}(t)|$  between the Hölder exponents of left and right engine where one of the two engines is fracture failure. And then, we calculate the Hölder exponents for are normal engine as shown in Fig. 4.

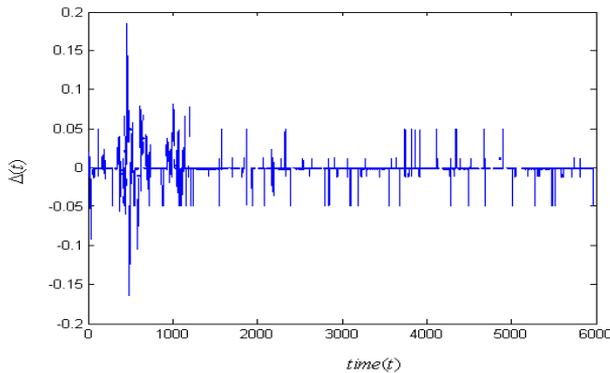
Fig. 5 indicates the error function for Hölder exponents in normal engine  $\Delta_n(t) = |h_{left}^{normal}(t) - h_{right}^{normal}(t)|$ , where  $h_{left}^{normal}(t)$ , and  $h_{right}^{normal}(t)$  are the Hölder exponent of normal left and right engine. Approximate to 0, compare to the nontrivial difference in Fig. 3.



**Fig. 3.** The error function  $\Delta_f(t)$  for one of the two engines is fracture failure in aircraft A



**Fig. 4.** The Hölder exponents of normal engine (a) left and (b) right for aircraft A



**Fig. 5.** The error function  $\Delta_n(t)$  of normal left and right engine for aircraft A.  
 $\Delta_f(t) \gg \Delta_n(t)$  indicate the Hölder exponent method to diagnose engine fault is available

### 3.2. Example 2: Aircraft B

To examine the availability of fractal comparative diagnostics method using data in aircraft B, we apply the Hölder exponent analysis method again.

Here, the Hölder exponent are calculated for aircraft B, as show in Fig. 6, where one of the engine is fracture failure. Obviously, there are different scaling phenomena between the Hölder exponents of left and right engine. For analysing the dissimilarity, the error function defined by Eq. (4) is employed once again.

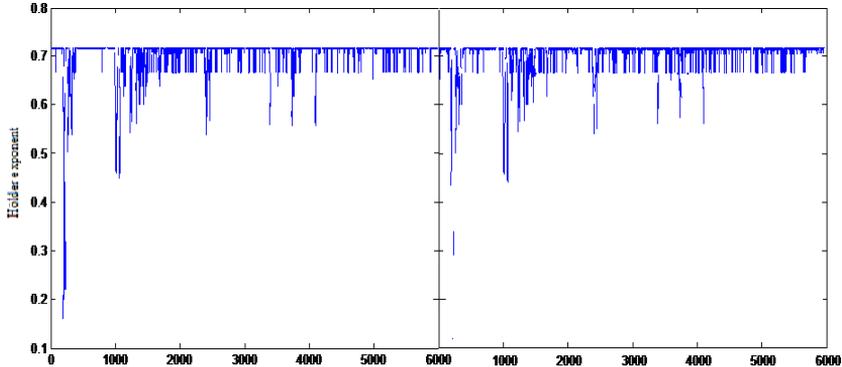


Fig. 6. The Hölder exponents of (a) left engine and (b) right engine for aircraft B where one of the engine is fracture failure

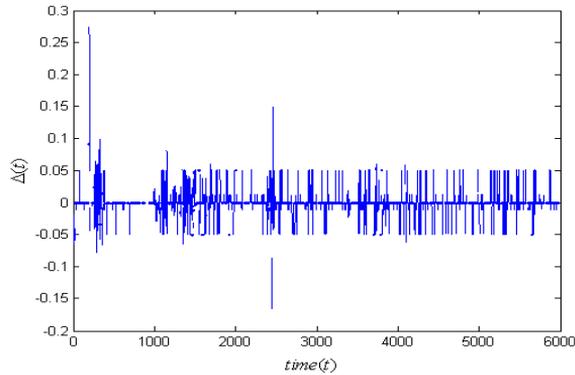


Fig. 7. The error function  $\Delta_f(t)$  for one of the two engines is fracture failure in aircraft B

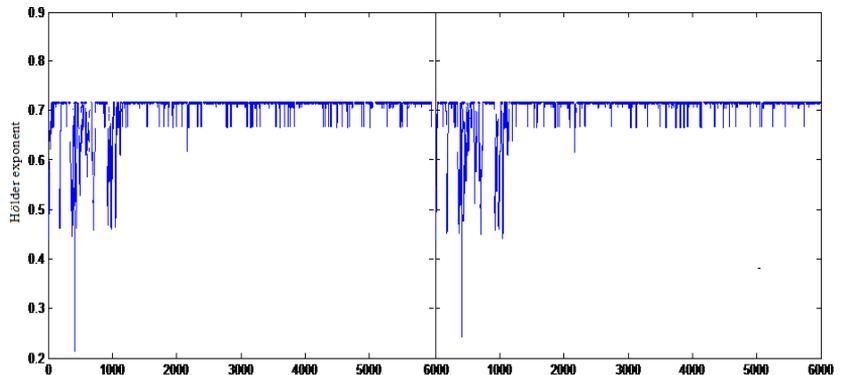


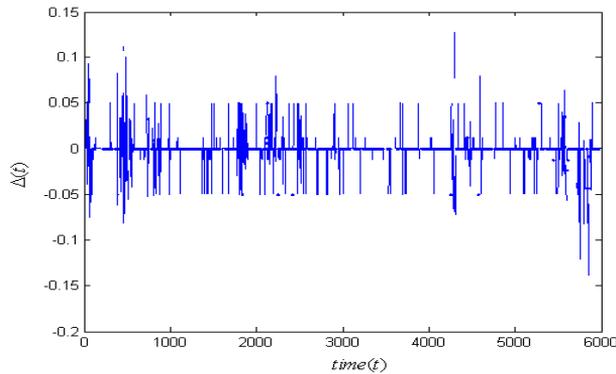
Fig. 8. The Hölder exponents of (a) left engine and (b) right engine for aircraft B for normal engine

We represent the error function  $\Delta_f(t) = |h_{left}(t) - h_{right}(t)|$  between the Hölder exponents of left and right engine in Fig. 7. For comparison, we calculate the Hölder exponents for a normal engine as shown in Fig. 8.

In Fig. 7, we observe that the nontrivial values of error function  $\Delta_f(t)$ , where one of the engines is fracture failure. For comparison, the error function:

$$\Delta_n(t) = |h_{left}^{normal}(t) - h_{right}^{normal}(t)|,$$

for normal engine are shown in Fig. 9. As expected the error function  $\Delta_n(t)$  is less than the error function  $\Delta_f(t)$  distinctly. We show the negligible difference between the Hölder exponents of left and right engine in Fig. 9, where both the left and right engine are normal.



**Fig. 9.** The error function  $\Delta_n(t)$  of normal left and right engine for aircraft B. The error function  $\Delta_f(t) \gg \Delta_n(t)$  indicate the Hölder exponent method is available for diagnosing engine fault

#### 4. Conclusions

We present a method of estimating the engine health and performance. The method is motivated by the local Hölder exponent theory, and implemented on the comparative analysis. Contrary to the intrinsically time-frequency analysis, this estimate is robust by providing stable, effective Hölder exponents, local in scale and position.

In this paper, we consider the effectiveness of the proposed method by applying it to time series collected at different aircraft. The conclusions indicate that the difference between left and right engine is obvious when one of the engines is fracture failure. On the other hand, the difference between the local Hölder exponents of left and right engine is trivial while two engines are normal. Therefore, we do believe that our monitoring algorithm may provide some help to analyze the aeroengine health and performance.

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