A Study of Inertia Force Identification by Inverse Method Between Cantilever Structure and Moving Mass

H Yan[1], Q Chen[2], G Yang[3], R Xu[4] and M Wang[5]
[1, 2, 5] Department of Control and System Engineering, Nanjing University, Nanjing, 210093, China
[3, 4] School of Mechanics, Nanjing University of Science and Technology, Nanjing, 210094, China

E-mail: chenqia0772@vip.sina.com

Abstract. In this paper, an inverse method which formed by two different parts: Kalman filter part and recursive least-square algorithm part is applied to solve an identification problem of the inertia force between a cantilever beam and a moving mass. Based on the basic Euler-Bernoulli beam model, the discretized state space model of the cantilever beam and moving mass system, which transform from the differential equations about the inertia force and modal responses of the cantilever beam, is established. Both the recursive inverse method and the traditional least square method are adapted to the none-noise and noise simulation deflection data which is obtained by the finite element method. The results show that although both two algorithms can bring good results in dealing with the none-noise data, the inverse method has a strong ability to estimate the inertia force in the strong-noise environment, where the traditional least square method fails. Finally, a field experiment is conducted and the identification results show that the recursive inverse method can be adapted to estimate the inertia force between the cantilever and moving mass successfully.

1. Introduction

The identification problem of the inertia force between the cantilever beam and moving mass is very important to many scientific and engineering fields such as structure engineering. By obtaining the accurate identification result, some important parameters of cantilever beam structure can be accessed to. However, if any sensor is put between cantilever beam and moving mass, it will damage the structure condition and the inertia force detected in this way won't be accurate. Therefore, a lot of indirect methods are applied to this identification problem.

A lot of methods to solve the inertia force or moving loads identification problems have been established by the former researchers and some of these methods are based on the classic Euler-Bernoulli beam model, such as IMI, IMII, FTDM and TDM [1-4]. Minzhuo Wang [5] modified the IMII, making it suitable to be applied to the inertia force identification problem between the cantilever beam and moving mass.

R E Kalman [6] proposed Kalman filtering technique in 1960 and P C Tuan [7] proposed an recursive inverse method, which consists of two parts: Kalman filter part and recursive least squares method part, to solve the input heat inverse estimation problems. C K Ma [8] adapted this recursive inverse method to solve some position static force identification problems in many different mechanical structures, such as beam structure and cantilever plate.

In this paper, the basic model proposed by Minzhuo Wang is modified, the discretized state space model of the cantilever beam and moving mass system is proposed and the recursive inverse method is applied to estimate the inertia force between them. The validity of combination of this identification model and method will be tested by the field experiments.

2. Mathematical model of the problem

2.1. Differential Equations of Modal Responses and Inertia Force of cantilever

The differential equation on the deflection of an Euler-Bernoulli beam is given as below [1]:

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[2] Corresponding author

$$\rho\frac{\partial^2 v(x,t)}{\partial t^2}+EI\frac{\partial^4 v(x,t)}{\partial x^4}=\delta(x-ct)f(t) \tag{1}$$

where ρ is a constant of mass per unit length of cantilever, $v(x,t)$ is the beam deflection at point x and time t, EI is a constant of flexural stiffness of cantilever, δ is the Dirac delta function, c is the speed of the mass, $f(t)$ is the time-varying inertia force. Because the interaction surface between the cantilever and moving mass is very smooth and is lubricated by lubricating oil, the damping of the motion process can be neglected.

After some calculation procedure [5], the differential equation of the modal response and time-varying inertia force can be obtained as:

$$\begin{bmatrix}\ddot{q}_{1,k}\\ \ddot{q}_{2,k}\\ \vdots\\ \ddot{q}_{n,k}\end{bmatrix}+\begin{bmatrix}\omega_1 q_{1,k}\\ \omega_2 q_{2,k}\\ \vdots\\ \omega_n q_{n,k}\end{bmatrix}=\frac{1}{\rho L}\begin{bmatrix}\varphi(1,l(k))\\ \varphi(2,l(k))\\ \vdots\\ \varphi(n,l(k))\end{bmatrix}f(t) \tag{2}$$

where

$$\omega_{(j)}^2=\frac{j^4\pi^4}{L^4}\frac{EI}{\rho},\quad j=1,2,3...$$

is the modal frequency of the simple support beam at the j th modal,

$$\cos\beta_i L\bullet ch\beta_i L=-1 \tag{3}$$

$$\varphi(i,x)=\mathrm{ch}\left(\frac{\beta_i\bullet x}{L}\right)-\cos\left(\frac{\beta_i\bullet x}{L}\right)-\frac{\mathrm{sh}(\beta_i)-\sin(\beta_i)}{\mathrm{ch}(\beta_i)+\cos(\beta_i)}\bullet\left(\mathrm{sh}\left(\frac{\beta_i\bullet x}{L}\right)-\sin\left(\frac{\beta_i\bullet x}{L}\right)\right) \tag{4}$$

are the frequency function and the modal equation of cantilever beam, respectively. L is the length of the cantilever beam, $q_{i,k}$ is the i th modal displacement and $\ddot{q}_{i,k}$ is the i th accelerate of the modal vibration of the cantilever. n is the maximum order of modal, $l(k)$ is the distance between the sampling point and the start point of the cantilever at k th sampling moment.

2.2. Discretized state function of the identification system

Equation (2) can be rewrite as follow:

$$\ddot{\mathbf{q}}+\boldsymbol{\omega}\mathbf{q}=\frac{1}{\rho L}\boldsymbol{\varphi}p \tag{5}$$

where $\ddot{\mathbf{q}}$ and $\mathbf{q}$ are the $n\times 1$ vector of the acceleration and displacement of modal vibration respectively, $\boldsymbol{\omega}$ is the $n\times n$ matrix of the modal frequency, $\boldsymbol{\varphi}$ is the $n\times 1$ matrix of modal function and p is the inertia force between cantilever beam and moving mass.

Equation (5) can be transferred to the discretized state model over time intervals of length Δt with process noise input as below:

$$\mathbf{X}(k+1)=\phi\mathbf{X}(k)+\Gamma[p(k)+w(k)], \tag{6}$$

$$\mathbf{Z}(k)=\mathbf{H}\mathbf{X}(k)+\mathbf{v}(k) \tag{7}$$

$$\mathbf{X}(k)=\begin{bmatrix}\mathbf{q}(k)\\ \dot{\mathbf{q}}(k)\end{bmatrix},\quad \phi=e^{\mathbf{A}\Delta t},\quad \Gamma=\int_{k\Delta t}^{(k+1)\Delta t}e^{\mathbf{A}[(k+1)\Delta t-\tau]}\mathbf{B}d\tau$$

$$\mathbf{A}=\begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{I}_{n\times n}\\ -\boldsymbol{\omega} & \mathbf{0}_{n\times n}\end{bmatrix},\quad \mathbf{B}=\frac{1}{\rho L}\begin{bmatrix}\mathbf{0}_{n\times 1}\\ \boldsymbol{\varphi}\end{bmatrix},\quad \mathbf{X}(t)=\begin{bmatrix}\mathbf{q}(t)\\ \dot{\mathbf{q}}(t)\end{bmatrix},$$

where $\mathbf{X}(k)$ is the state vector of the system, ϕ is the state transition matrix, Γ is the input matrix, $p(k)$ is the inertia force between cantilever and moving mass which is going to be estimate and $w(k)$ is the input system noise which is assumed to be zero mean and white with variance $E\{w(k_1)w(k_2)\}=Q\delta_{k_1k_2}$, Q is the process noise covariance and $\delta_{k_1k_2}$ is the Dirac delta function. $\mathbf{H}=\mathbf{I}_{2n\times 2n}$ is the measurement matrix, $\mathbf{Z}(k)=[Z_1(k)\,Z_2(k)\cdots Z_{2n}(k)]^T$ is the observation vector and the measurement noise vector $\mathbf{v}(k)=[v_1(k)\ v_2(k)\cdots v_{2n}(k)]^T$ is assumed to be zero mean and white. The variance of $\mathbf{v}(k)$ is $E\{\mathbf{v}(k_1)\mathbf{v}^T(k_2)\}=\mathbf{R}\delta_{k_1k_2}=diag\left[\sigma_1{}^2\ \ \sigma_2{}^2\ \dots\ \sigma_{2n}{}^2\right]\delta_{k_1k_2}$ and the elements σ_i is the standard deviation of measurement noise of $v_i(k)$.

2.3. Recursive algorithm of inertia force identification

The recursive identification algorithm, which is shown below in table 1, consists of two parts: Kalman filter part, which is adapted to calculate the residual innovation sequence, and recursive least-square estimation part, which estimates the inertia force between cantilever and moving mass.

Table 1. The recursive identification algorithm.

The Kalman filter part	The recursive least-square estimation part
$\overline{\mathbf{X}}(k/k-1)=\phi\overline{\mathbf{X}}(k-1/k-1),$ (8)	$\overline{\mathbf{B}}(k)=\mathbf{H}[\phi\mathbf{M}(k-1)+\mathbf{I}]\boldsymbol{\Gamma},$ (15)
$\mathbf{P}(k/k-1)=\phi\mathbf{P}(k-1/k-1)\phi^T+\Gamma Q\Gamma^T,$ (9)	$\mathbf{M}(k)=[\mathbf{I}-\mathbf{K}(k)\mathbf{H}][\phi\mathbf{M}(k-1)+\mathbf{I}],$ (16)
$\mathbf{S}(k)=\mathbf{H}\mathbf{P}(k/k-1)\mathbf{H}^T+\mathbf{R},$ (10)	$\mathbf{K}_b(k)=\gamma^{-1}\mathbf{P}_b(k-1)\overline{\mathbf{B}}^T(k)\bullet[\overline{\mathbf{B}}(k)\gamma^{-1}\mathbf{P}_b(k-1)\overline{\mathbf{B}}^T(k)+\mathbf{S}(k)]^{-1},$ (17)
$\mathbf{K}(k)=\mathbf{P}(k/k-1)\mathbf{H}^T\mathbf{S}^{-1}(k),$ (11)	$\mathbf{P}_b(k)=[\mathbf{I}-\mathbf{K}_b(k)\overline{\mathbf{B}}(k)]\gamma^{-1}\mathbf{P}_b(k-1),$ (18)
$\mathbf{P}(k/k)=[\mathbf{I}-\mathbf{K}(k)\mathbf{H}]\mathbf{P}(k/k-1),$ (12)	$p(k)=p(k-1)+\mathbf{K}_b(k)[\mathbf{Z}(k)-\overline{\mathbf{B}}(k)p(k-1)],$ (19)
$\overline{\mathbf{Z}}(k)=\mathbf{Z}(k)-\mathbf{H}\overline{\mathbf{X}}(k/k-1),$ (13)	
$\overline{\mathbf{X}}(k/k)=\overline{\mathbf{X}}(k/k-1)+\mathbf{K}(k)\overline{\mathbf{Z}}(k).$ (14)	

where $\overline{\mathbf{X}}(k/k-1)$ is the state prediction without considering the system input $p(k)$, $\overline{\mathbf{X}}(k/k)$ is the state updated estimation, $\mathbf{P}(k/k-1)$ is the covariance of state prediction, $\mathbf{P}(k/k)$ is the updated state covariance, $\mathbf{K}(k)$, $\mathbf{S}(k)$ and $\overline{\mathbf{Z}}(k)$ are the Kalman gain, covariance and innovation, respectively, $\mathbf{K}_b(k)$ is the gain of recursive least-square algorithm, $\mathbf{P}_b(k)$ is the error covariance of the estimation of inertia force, $\overline{\mathbf{B}}(k)$ and $\mathbf{M}(k)$ are the sensitivity matrices.

The initial parameters of the recursive inverse method are set as below [9]:

$$\overline{\mathbf{X}}(-1/-1)=[0\,0\cdots 0]^T,\quad p(-1)=0,\quad \mathbf{M}(-1)=\mathbf{0}_{n\times n},\quad \mathbf{P}(-1/-1)=diag[10^{10}],\quad \mathbf{P}_b(-1)=diag[10^9]$$

$$Q=10^{-4},\quad \mathbf{R}=diag\left[10^{-3}\,10^{-5}\,10^{-6}\,10^{-6}\,10^{-2}\,10^{-4}\,10^{-5}\,10^{-5}\right]$$

As it mentioned in[7], the scalar parameter γ is very important to the performance of the algorithm. Because the vibration data of the cantilever beam doesn't vary fast and it will be affected by the measurement noise, the scale parameter γ is set as 0.4.

The detailed derivation of this estimation algorithm can be found in Tuan's work [7].

3. Field Experiment

In order to verify the feasibility of the recursive inverse method besides the simulation results [9], a field experiment is designed and conducted.

3.1. Experiment setup

As it shown in figure 1, a set of experimental apparatus consists of cantilever and moving mass are build up. The cantilever parameters are: the length L is 1.5m, the constant flexural stiffness EI is $3.42\cdot10^5$Nm^2 and the constant mass per unit length ρ is $7.8\cdot10^3$kg/m^3. The sampling frequency is 10KHz. A nitrogen propulsion unit is designed to provide the initial impact to the moving mass. Ten laser displacement sensors (Keyence, LK-G400) are evenly placed under the cantilever beam that the length between each of two sensors is 150mm. The measurement noise of the laser displacement sensor is tested before the experiment and the standard deviation of its measurement noise is approximately equal to 10^{-7}. Data acquisition instrument (Dewetron 1201) is used to provide the sync signals to the laser displacement sensors and record the deflection data of cantilever.

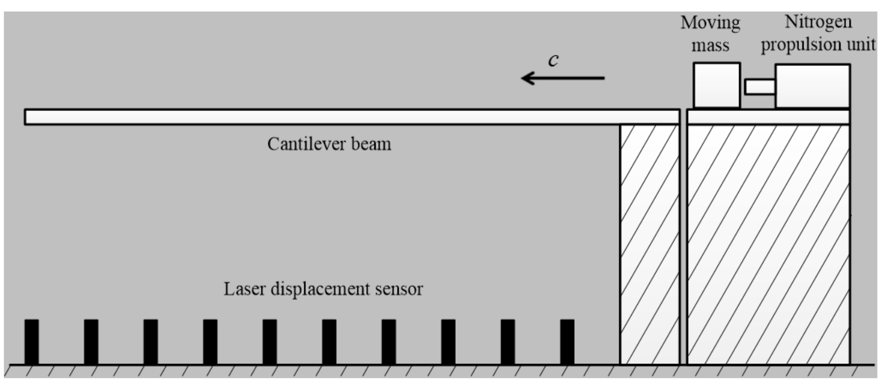

Figure 1. The experiment schematic diagram.

The contact surfaces between the cantilever beam and moving mass are manufactured very smooth and the lubricating oil is added between the surfaces, therefore, the moving damp can be neglected and the velocity of the moving mass can be assumed to be constant. In fact, in the prep experiments, a high speed camera (IDT-Y3) is set up to record the motion process of the mass and a motion analysis software (ProAnalyst) is used to calculate the speed of the moving mass. The results show that the decrease amount of the speed is very small and therefore the model assumption that the damp can be neglected is appropriate.

As soon as the mass moves on the cantilever, the laser displacement sensors begin to record the displacement of the cantilever and a high speed camera (not be shown in figure 1), which is synchronous with the data acquisition device, is set at the fixed end of the cantilever to record the exact moment the mass move out the fixed end of the cantilever beam which can be seen as the start signal of the data recording.

3.2. Experiment results

Three experiments with different mass velocity, c=10m/s, 15m/s and 20m/s, are conducted and the identification result of the inertia force of each experiment is shown in figure 2-4.

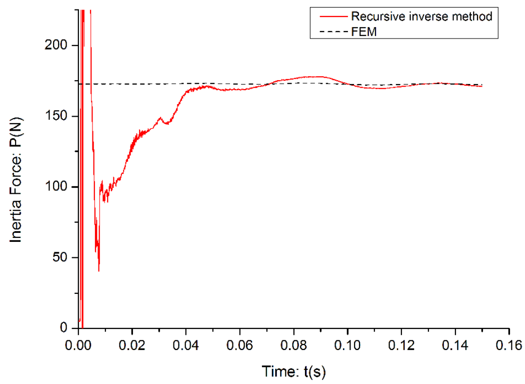

Figure 2. The experiment identification result (c=10m/s).

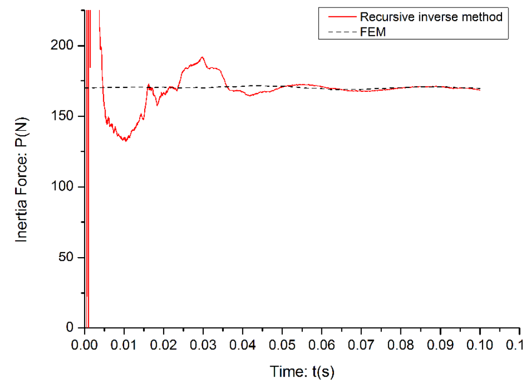

Figure 3. The experiment identification result (c=15m/s).

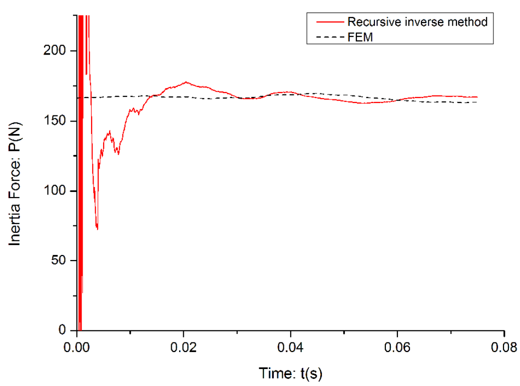

Figure 4. The experiment identification result (c=20m/s).

As it can be seen in the figure 2-4, the initial part of the identification process is very unstable and it is obvious not caused by the signal noise or instability of the algorithm. After checking the deflection data, it is found that when mass has just moved on the cantilever, it will bring an impact to the structure and even when it pass the fixed end, the impact wave can't vanish in such short time and it will interfere the deflection of cantilever beam. After the impact wave vanish in a short period, the recursive inverse method can identify the inertia force between cantilever and moving mass very accurately.

For evaluating the effectiveness of the algorithm, a criterion called relative percentage error (RPE) [2] is introduced as follow:

$$\text{RPE}=\frac{\sum\left|f_{ture}-f_{ident}\right|}{\sum\left|f_{ture}\right|}\times 100\% \tag{20}$$

The RPE of the stable part of each identification result is listed in table 2.

Table 2. The RPE of the experiment identification results

Velocity of moving mass (m/s)	RPE
10	1.145
15	1.247
20	1.739

4. Discussion

In the field experiments, the initial part of the identification process is very unstable because when the mass has just moved on the fixed part of cantilever, the impact wave can't be vanished before it moves out the fixed end of the cantilever. If the fixed part can be extended, moving mass will be able to stay in the fixed part of the cantilever for a longer time and the impact wave can be vanished which will be helpful to the identification.

The whole identification model is based on the assumption that the velocity of the moving mass is stable and as mentioned in section 3.1, it has been proved by the prep experiment result that our experiment apparatus meet this assumption. However, if the length of the cantilever beam is much longer or contact surface are not smooth enough, this assumption can't be correct because the mass will lose some energy in the process of moving and the identification model should be modified in that case.

In order to obtain the deflection data of the cantilever beam, the sensors should be chosen very carefully. The sensors should have a high resolution because the deflection of the cantilever beam is very small. What's more, the sensors should be very stable and the measurement noise of the signal can't be very severe.

5. Conclusion

In this paper, the recursive inverse method is adapted to identify the inertia force between the cantilever beam and moving mass. A new identification model is formed and an experiment is set up to obtained the deflection data of the cantilever. The feasibility of the identification model and method is verified by the field experiment results.

Future work will focus on the improving of the experiment setup and the identification method itself.

Acknowledgments

This work was supported in part by the Major State Basic Research Development Program of Republic of China (No. 613116).

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