839. Vibration analysis of geared rotor system under time varying mesh stiffness effects

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Abstract. The present work contributes to the analysis of the interactions between gears, shafts and journal bearings in a geared rotor-bearing system. Although there are analyses for both of the gear and rotor-bearing system dynamics, the coupling effect of the nonlinear variable pressure angle and geared rotor-bearing system is deficient. In contrast to the majority of the models in the literature, the variable mesh stiffness and pressure angle are introduced in this paper while they were considered as constant in previous models. The equations of motion for the geared rotor-bearing system are obtained by applying Lagrange’s equation, and the Runge-Kutta numerical method is used to solve the equations of motion. Numerical results of this study indicated that the proposed model provides realistic dynamic response of a geared rotor-bearing system.

Keywords: gear, rotor system, time varying mesh stiffness.

1. Introduction

The geared rotor-bearing system is one of the main mechanisms for modern power transmission. It is often coupled with bearing systems, power generation and power extraction. On account of the increasing demand for high speed and accurate transportation, the research in the field of geared rotor dynamics is very important. The dynamic analysis of the geared rotor-bearing system with variable pressure angle is investigated in this paper.


2. Modeling of the System

The configuration of a geared rotor-bearing system is shown in Fig. 1. Two uniform flexible shafts are of length \( L_1 \) and \( L_2 \), and the gear pair is mounted on the shafts. An external torque \( M \) exerts upon the driving gear. The contacting mesh force is represented by the gear mesh stiffness \( k_m \) and damping \( c_m \) along the pressure line. Four bearings are modeled as flexible elements with damping and stiffness denoted as \( c_j^b \) and \( k_j^b \). A single shaft system with a rigid disk is shown in
Fig. 2, in which the fixed reference frame, $X-Y-Z$, is used to describe the system motion. Five degrees of freedom $V$, $W$, $\alpha$, $B$, $\Gamma$ are considered at each nodal point of the shaft, where $V$ and $W$ are lateral displacements along $Y$ and $Z$ directions, respectively, $B$ and $\Gamma$ are rotational displacements, $\alpha$ is the torsional displacement, and the axial translational vibration is neglected.

![Fig. 1. The configuration of a geared rotor-bearing system](image)

![Fig. 2. Typical rotor configuration and coordinates](image)

Fig. 3 shows the configuration and generalized coordinates for the pair of gears [15], which are mounted on the shafts as shown in Fig. 1. The teeth are regarded as flexible cantilever beams and are deformed by both the bending and shear. The mass, transverse mass moment of inertia, polar mass moment of inertia of the driving gear and driven gear are, respectively, $m_{d1}$, $I_{d1x}$, $I_{d1y}$ and $m_{d2}$, $I_{d2x}$, $I_{d2y}$. The motion of the gear set can be defined with ten generalized coordinates, which are the same as the node coordinates of two shafts, where the gears are mounted. The dashed and solid circles represent the gear pair before and after motion, respectively. $O_1$ and $O_2$ are the initial centers of each shaft, and $C_1$ and $C_2$ are the axis centers after the motion. The mass centers of gears are denoted by $G_1$ and $G_2$. The magnitudes of the eccentricity denoted by $e_1$ and $e_2$, and the eccentricity angles are denoted as $\Phi_{d1}$ and $\Phi_{d2}$. The angular coordinates of the driving and driven gears are defined as:
\[ \theta_i(t) = \pm \Omega_i t + \alpha_{d_i}(t) + \Phi_{d_i} \]  

(1)

where \( i = 1 \) and 2 mean the driving and driven gears, respectively. \( \Omega_i \) are the spin speeds of shaft. The displacement vectors for the mass centers of the gears are defined as:

\[ \mathbf{r}_i(t) = [V_{ai}(t) + e_i \cos \theta_i(t)] \hat{j}_i + [W_{ai}(t) + e_i \sin \theta_i(t)] \hat{k}_i \]  

(2)

where \( \hat{j}_i \) and \( \hat{k}_i \) are the unit vectors in \( X_i \) and \( Y_i \) axes. The distance between centers, \( d \), is changed to \( d' \) after the motion, and \( d' \) is defined as:

\[ d'(t) = \sqrt{[V_{d2}(t) - V_{d1}(t) + d]^2 + [W_{d2}(t) - W_{d1}(t)]^2} \]  

(3)

Fig. 3. The configuration and generalized coordinates for gear pair

In Fig. 4, the gear pair is modeled as the equivalent stiffness \( k_m \) and damping \( c_m \) with transmission error \( e_m(t) \) along the pressure line between the teeth. The equivalent stiffness and damping are treated as time-varying coefficients in this paper. The pressure line is defined as the common tangent line of the base circles for the gear set, and the pressure angle \( \phi_p \) is defined as:

\[ \phi_p(t) = \cos^{-1}[(R_1 + R_2) / d'(t)] \]  

(4)

where \( R_1 \) and \( R_2 \) are the radii of base circles of gears.

The angle of the driven gear relative to the driving gear is represented by:

\[ \psi(t) = \tan^{-1}[(W_{d2} - W_{d1})/(V_{d2} - V_{d1} + d)] \]  

(5)

The pressure angle and the gear position angle in this paper are affected by the translational motions of the gear pair while the previous studies neglected these effects. According to the above assumption that the gear mesh is modeled as the equivalent stiffness and damping with transmission error, the gear mesh deformation along the pressure line can be written as:
\[ \delta(t) = (V_{d2} - V_d)\sin(\phi_p - \psi) + (W_{d2} - W_d)\cos(\phi_p - \psi) - (R_1\alpha_{d1} + R_2\alpha_{d2}) + e_t \]  \hspace{1cm} (6)

\[ \theta(t) = \Omega z\frac{t}{2} + \alpha_{d1}(t) + \Phi \]  \hspace{1cm} (7)

\[ \delta(t) = \Omega z\frac{t}{2} + \alpha_{d2}(t) + \Phi \]  \hspace{1cm} (8)

\[ m_p(t) = \left[ \sqrt{A_1^2 - R_1^2} + \sqrt{A_2^2 - R_2^2} - d\sin\phi_p(t) \right] / p_b \]  \hspace{1cm} (9)

3. Governing Equation of the System

The components of this geared rotor-bearing system include the gear pairs, bearing supports and rotor shafts. The equations of motion are derived for each component as follows.

The gear pair in this paper is assumed as two disks, so the kinetic energies of driving gear and driven gear can be described as:

\[ T_i^r = \left[ m_\omega \hat{\delta}_i^2 + I_{d_1} (\pm \Omega z + \dot{\phi}_d)^2 + I_{a_1} \hat{\theta}_a^2 + I_{a_1} \hat{\gamma}_a^2 + I_{d_2} (\pm \Omega z + \dot{\phi}_d)(\dot{B}_a \hat{\gamma}_a - \hat{\gamma}_a \dot{B}_a) \right] / 2 . \]  \hspace{1cm} (10)
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Fig. 5. Non-linear stiffness model with mesh period

The components of the gear mesh forces on the driving gear and the moment about the geometric center can be expressed:

\[ F_{p1} = -F_r \sin(\phi_p - \psi) = -(k_n \delta + c_n \dot{\delta}) \sin(\phi_p - \psi) \]  

\[ F_{p2} = -F_r \cos(\phi_p - \psi) = -(k_n \delta + c_n \dot{\delta}) \cos(\phi_p - \psi) \]  

\[ T_{k1} = -R_k F_m = -R_k (k_n \delta + c_n \dot{\delta}) + M \]

For the driven gears, similar expressions can be obtained by applying the Lagrange approach and neglecting the transmission error, the equations of motion of the gear pair can be obtained:

\[ \begin{bmatrix} M_1 \end{bmatrix} \ddot{q}_d + \left( \begin{bmatrix} C_1 \end{bmatrix} + c_n \begin{bmatrix} K_1 \end{bmatrix} \right) \dot{q}_d + \left( k_n \begin{bmatrix} K_1 \end{bmatrix} + c_m \Phi \begin{bmatrix} K_2 \end{bmatrix} \right) q_d = \begin{bmatrix} F_d \end{bmatrix} \]  

\[ \text{where} \quad \{q_d\} = \begin{Bmatrix} V_{d1} \ W_{d1} \ B_{d1} \ \Gamma_{d1} \ \alpha_{d1} \ V_{d2} \ W_{d2} \ B_{d2} \ \Gamma_{d2} \ \alpha_{d2} \end{Bmatrix} \]  

For shafts, two-noded element is used for the finite element formulation in this paper. As is shown in Fig. 6, five degrees of freedom are considered at each nodal point. The kinetic energy and potential energy of the shaft element can be expressed as:

\[ T^e = 1/2 \int \left( \rho A (V_z^e + \dot{W}_z^e)^2 + I_{\alpha} (\ddot{B}^e_z + \ddot{\Gamma}^e_z)^2 + I_{\alpha'} (\Omega_z + \dot{\alpha}_z)^2 + I_{\alpha'} (\Omega_z + \dot{\alpha}_z)(\dot{B}_z \Gamma_z - \dot{\Gamma}_z B_z) \right) ds \]  

\[ U^e = 1/2 \int \left[ EI (B_z')^2 + (\Gamma_z')^2 \right] ds + \kappa' G A [(V_z' - \Gamma_z')^2 + (W_z' + B_z')^2] ds \]

\[ \text{where} \quad \rho, A, I, I_{\alpha}, I_{\alpha'}, E, G \text{ and } \kappa' \text{ are the parameters and modulus of the shafts[12]. By Lagrange approach, the motion equations of shaft elements can be obtained:} \]

\[ \{M^e\} \ddot{q}_e^e + \Omega[G^e] \{\dot{q}_e^e\} + \{K^e\} \{q_e^e\} = 0 \]

\[ \text{where} \quad \{q_e^e\} = \begin{Bmatrix} V_{e1} \ W_{e1} \ B_{e1} \ \Gamma_{e1} \ \alpha_{e1} \ V_{e2} \ W_{e2} \ B_{e2} \ \Gamma_{e2} \ \alpha_{e2} \end{Bmatrix} \]

In this paper, those four bearing supports are assumed as isotropic. The potential energy and virtual work of \( j \)th bearing support can be expressed as:

\[ U^e_j = (k_j^b V_{b_j}^2 + k_j^b W_{b_j}^2)/2 \quad \text{and} \quad \delta W^e_{j} = -c_j^b \dot{V}_{b_j} \delta V_{b_j} - c_j^b \dot{W}_{b_j} \delta W_{b_j} \]
By the Lagrangian approach, the motion equations of the \( j \)th bearing support can be obtained as:

\[
\begin{bmatrix}
    c_j^b & 0 \\
    0 & c_j^b
\end{bmatrix}
\begin{bmatrix}
    \ddot{q}_j^b
\end{bmatrix} +
\begin{bmatrix}
    k_j^b & 0 \\
    0 & k_j^b
\end{bmatrix}
\begin{bmatrix}
    q_j^b
\end{bmatrix} =
\begin{bmatrix}
    0 \\
    0
\end{bmatrix}
\]

(19)

where \( \{q_j^b\} = \{V_{ij}, W_{ij}\} \) and \( j = 1 \sim 4 \).

The system equation of motion can be obtained by assembling equations of each component in the above, and is expressed as:

\[
[M']\ddot{q}^s + [C']\dot{q}^s + [K']q^s = \{F^s\}
\]

(20)

where \( \{q^s\} \) is the state vector, \([M']\), \([C']\) and \([K']\) are the system mass, damping and stiffness matrices, respectively. \( \{F^s\} \) is the forcing vector due to the mesh, external torque and eccentricity effects, which includes the non-linear mesh effect.

4. Results and Discussion

In order to compare the results of Kim et al. [15], the eccentricity of the gear is neglected and the torsional bearing dampings \( c_i^b \), \( c_2^b \) are considered. The parameters of the gear pair and rotor system are shown in refs. [12] and [15]. The rotating speeds of the shafts are considered as constants in this paper, and the acceleration process is neglected. Fig. 7 shows the gear mesh deformations with different rotating speeds when both of the shaft lengths are zero. The equivalent damping \( c_{j, equl}^b \) and stiffness \( k_{j, equl}^b \) in each bearing are \( c_{j, equl}^b = 2c_j^b \), \( k_{j, equl}^b = 2k_j^b \). The amplitudes of those deformations in Fig. 7 are similar to those in Kim et al. [15], and this indicates the accuracy of the simulation in this paper. The details of the deformations are shown in Fig. 8. The differently rotating speeds change the frequency of mesh, and the contact ratio of the mesh is also varied. But the varying of amplitude is unobvious. Because the eccentricity is neglected that the lateral deformations \( r_1 \), \( r_2 \) are not effected as the varying rotating speed varies.

In the followings, the shaft lengths are considered for \( L=254 \) as Shiau's [12], and \( \Omega_1 \) is
selected as 3000 rpm. Fig. 9 shows the lateral deformation $r_i$ and gear mesh deformations with/without the rotating shaft. In the above figures, the equivalent damping $c_{i,\text{equi}}^b$ and stiffness $k_{j,\text{equi}}^b$ are selected as $c_{1,\text{equi}}^b = 5.6$, $c_{2,\text{equi}}^b = 8.4$ (Ns/m) and $k_{1,\text{equi}}^b = 106$ (N/m). The solid lines in Fig. 9 and Fig. 10 mean the geared rotor-bearing system with shaft $L=254$, and the dashed lines mean the shaft $L=0$ respectively. In this paper the damping and stiffness coefficients, $c_j^b$ and $k_j^b$, are assumed as the same value, $c_b$ and $k_b$. As the shown in Fig. 9(a), the system with shaft $L=254$ is not convergent before 0.02 sec. as the damping is selected as 2.8 and 4.2. (The equivalent damping coefficients $c_{1,\text{equi}}^b$, and $c_{2,\text{equi}}^b$ are 5.6 and 8.4 as in Kim et al. [15].) Fig. 9(a) also shows the lateral deformation with different damping coefficients, which effects the amplitude and convergence time for the vibration of lateral deformation. This geared rotor-bearing system with shaft $L=254$ can be convergent effectively same as the system $L=0$ when the damping coefficients are selected as $c_b = 800$ (Ns/m). It means if the time-varying gear mesh stiffness and pressure angle in 2-D plane as Kim et al. [15] are considered into the 3-D geared rotor-bearing system in this paper, the damper should be redesigned. Fig. 9(b) shows the detail of the gear mesh deformation. The frequency content of responses is more complex when the time-varying mesh effect is considered as the system in this paper. However, as shown in Fig. 9(b), the mesh periods $T_m$ are the same for system with/without the rotating shaft.

Fig. 10 shows the lateral deformations $r_i, r_2$ with different bearing stiffness. In Fig. 10, damping coefficients $c_j^b$ are selected as 1000 Ns/m. Comparing the dashed lines and solid lines in Fig. 10(a) and Fig. 10(b) with $k_{1,\text{equi}}^b = k_{2,\text{equi}}^b = 10^6$ N/m and $k_b = 5 \times 10^5$ N/m, the magnitudes of lateral deformations in steady state are increased when the rotating shaft $L=254$ is considered. For the reason that the shafts are flexible, this will increase the deformations of the gears. Especially while the gear pair is mounted at the middle of the shaft. In Fig. 10(a) and Fig. 10(b), increasing the bearing stiffness can reduce the magnitudes of lateral deformations $r_i, r_2$ in steady state effectively.
Fig. 8. The detail deformations with different rotating speeds

Fig. 9. Lateral deformations $r_1$ and gear mesh deformations with/without the rotating shaft.
5. Conclusions

The geared rotor-bearing system is one of the main mechanisms for modern power transmission. This paper investigates the dynamic responses by considering the couple effect of the time-varying mesh and geared rotor-bearing system, for it is deficient in the previous researches. The time-varying mesh effects include the pressure angle and contact ratio while the previous model has regarded them as constants. As the results show, the damper coefficient should be redesigned when this effect of time-varying gear mesh in 2-D plane are considered into the 3-D geared rotor-bearing system. Moreover, the mesh periods $T_m$ are the same for systems with/without the rotating shaft.

References

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