# 793. Instantaneous frequency identification of a time varying structure using wavelet-based state-space method

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**Abstract.** This paper presents a method to identify the instantaneous frequency of the time varying structures based on wavelet and state space methods by using free and forced vibration response data. Firstly, the second-order vibration differential equations are rewritten as the first-order state equations using state space theory. Secondly, both excitation and response signals are projected by the Daubechies wavelet scaling functions. Thus, the first-order state equations are transformed into linear algebraic equations using the orthogonality of the scaling functions. Lastly, the equivalent time varying state space system matrices of time varying structure are extracted directly by solving the linear equations. The instantaneous frequencies are determined via eigenvalue decomposition of the state space system matrices. The proposed identification algorithm is investigated with a four degrees-of-freedom spring-mass-damper model. Numerical simulations demonstrate that the proposed method is robust and effective for identification of the abruptly, smoothly and periodically changing instantaneous frequencies of time varying structures.

**Keyword:** time varying structure, instantaneous frequency, identification, wavelet, state space.

## 1. Introduction

A number of approaches have been proposed for the dynamic parameters identification of linear systems in the literature over the past 30 years [1-4]. The input-output modal identification methods are developed based either on a set of Frequency Response Functions or on the corresponding Impulse Response Functions. For very large and flexible structures like bridge or dam, the forced excitation requires extremely heavy and expensive equipment very seldom available in most dynamic experiments. The output-only modal identification approaches have been developed by estimation of Power Spectral Density Functions or Correlation Functions of ambient excitation response data. Reference [5] presents a brief review of the evolution of Experimental Modal Analysis from Input-Output to Output-Only modal identification techniques and their applications in civil engineering. Some benchmark problems of civil engineering structures for the system identification and damage detection of civil structures have been established [6-7]. Many international participants have proposed different approaches to identify structural damage using different modal identification techniques.

The Fourier transform (FT) can be considered as a decomposition of a signal into a linear combination of vectors given by Fourier coefficients. However this decomposition does not give time-localized information about the signal. The techniques based on the wavelet transform (WT) for structural damage detection have received considerable attention in recent years [8-9] because of their ability to decompose the measured signals in the frequency–time domain. The modal identification methods have been proposed to identify modal parameters base on wavelet analysis processing free responses of mechanical structures. The methods for estimating modal frequency and damping ratios have been proposed based on the continuous wavelet transform in references [10-11]. Reference [12] chose the Cauchy wavelet and developed a procedure to identify natural frequencies and viscous damping ratios as well as mode shapes. Reference [13] makes use of the continuous wavelet transform to develop a complete procedure for identification of natural frequency, viscous damping ratio and mode shapes from free responses. The choice of the mother wavelet and its localization properties is discussed. The edge effect is highlighted by introduction of two real coefficients to allow this effect to be taken in to account during the identification process.

The Hilbert transform (HT) also has the capability of decomposing measured signals in the frequency–time domain to extract the instantaneous frequency at every time instant. Therefore system identification methods based on the HHT are then developed to estimate the modal parameters of multi-degree-of-freedom (MDOF) systems including not only the natural frequencies, damping ratios, but also the mode shapes as well as the mass, stiffness and damping matrices of the linear systems in which the mode shapes may be real or complex [14-15]. Further, the Hilbert–Huang spectral analysis has been investigated for the health monitoring of structures and the damage detection of a benchmark problem.

In light of the above-mentioned references, all modal identification approaches and their application in damage detection are developed only for the linear time-invariant (LTI) systems. However, structural systems accumulate damage under service load and environmental excitations. In such case, a linear time-varying (LTV) model should be able to capture the instantaneous characteristics of the system and to identify its damage [16-17].

The identification of linear time-varying systems has received increasing attention nowadays. Some efforts have been made in extending the discrete time state-space identification algorithm from linear time-invariant systems to linear time-varying systems from the 90s [18-19]. A subspace-based identification algorithm that uses free responses to identify successive discrete transition matrices and use eigenvalues of the estimated transition matrices to characterize properties of the linear time-varying systems has been proposed [20]. Experimental verification studies on an axially moving cantilever beam were addressed in reference [21]. Reference [22] proposes a new adaptive tracking technique based on the least-squares estimation approach to track the abrupt change of the time-varying structural parameters. The wavelet-based identification approach for the analysis of linear time-varying systems was presented [23]. The identification algorithm focuses on the identification of the damping and stiffness parameters associated with the differential equation model relating input and the output measurements, which has been discretized following a Galerkin procedure using wavelet based [24]. The Hilbert transform technique has been investigated to identify system instantaneous modal parameters for nonlinear systems. For MDOF structures, an identification algorithm of linear time-varying system based on the Hilbert Transformation and empirical mode decomposition has been developed using free and forced vibration response data [25]. The proposed technique is capable of tracking slow, abrupt and periodic changes of damping and stiffness coefficients of systems.

In this paper, we propose a wavelet-based state-space identification approach for time-varying structures. We model the LTV structure using the state space method and extract the instantaneous frequency by wavelet analysis. The Daubechies wavelet analysis is introduced to the state space model, thus the first-order state equations can be transformed into linear algebraic equations using the orthogonality of the scaling functions and the time varying dynamic parameters can be directly estimated via solving the linear algebraic equations. More importantly, this improvement of combined use of two methods (wavelet and state-space) successfully avoids computing the connection coefficient between wavelet scaling functions and the derivative of wavelet scaling functions both of the first and second order. This improvement is able to save the computation time for the identification procedure.

## 2. State-space model of LTV structures

The vibration equation of a $p$ degrees-of-freedom LTV structure can be expressed as:

$$M(t)\ddot{x}(t)+E(t)\dot{x}(t)+K(t)x(t)=bu(t) \tag{1}$$

in which $M(t)$, $E(t)$ and $K(t)$ are $p\times p$ time-dependant mass, damping and stiffness coefficients matrices of the system respectively, $x(t)$ is the $p\times 1$ displacement vector, $b$ is the $p\times n_i$ input shape matrix, $u(t)$ is the $n_i\times 1$ input force vector, and $n_i$ denotes the number of input force signals. The output equation for the same LTV structure can be represented as:

$$y(t)=C_a\ddot{x}(t)+C_v\dot{x}(t)+C_d x(t) \tag{2}$$

where $y(t)$ is the $n_0\times 1$ output vector, which can be a combination of different types of responses, $C_a$, $C_v$, $C_d$ are the output matrices for the measurement of acceleration, velocity and displacement, respectively, $n_0$ is the number of output signals. Equations (1) and (2) can be transformed into the state space equation as follows:

$$\dot{z}(t)=A(t)z(t)+B(t)u(t) \tag{3}$$

$$y(t)=C(t)z(t)+D(t)u(t) \tag{4}$$

in which $z(t)$ is the $n_0\times 2p$ state vector, $A(t)$ is the $2p\times 2p$ system matrix, $B(t)$ is the $2p\times n_i$ input matrix, $C(t)$ is the $n_0\times 2p$ output influence matrix and $D(t)$ is the $n_0\times n_i$ direct transmission matrix, respectively. They are expressed as:

$$A(t)=\begin{bmatrix} 0 & I \\ -M^{-1}(t)K(t) & -M^{-1}(t)E(t)\end{bmatrix} \tag{5}$$

$$B(t)=\begin{bmatrix}0 & M^{-1}(t)b\end{bmatrix}^T \tag{6}$$

$$C(t)=\begin{bmatrix} C_d-C_aM^{-1}(t)K(t) \\ C_v-C_aM^{-1}(t)E(t)\end{bmatrix}^T \tag{7}$$

$$D(t)=C_aM^{-1}(t)b \tag{8}$$

## 3. Signal project using the Daubechies wavelet

Daubechies wavelet, with the vanishing moments of $N$, could be called $dbN$ wavelet. The state vector $z(t)$ can be projected by using scaling functions of $dbN$ wavelet at resolution $j$:

$$z(t)=\sum_{\mathrm{k}}\widetilde{\alpha}_k\cdot 2^{j/2}\phi(2^jt-k) \tag{9}$$

where integers $k \in Z$ are translation factors, $\tilde{\alpha}_k$ are the approximation coefficients, $\phi_{j,k}(t) = 2^{j/2}\phi(2^j - k)$ are Daubechies scaling functions. Let $r = 2^j t$ , $\alpha_k = \tilde{\alpha}_k 2^{j/2}$ and equation (9) is rewritten as follows:

$$z(t) = \sum_k \alpha_k \phi(r-k) = Z(r) \tag{10}$$

The first derivative of scaling function to time $t$ is:

$$\frac{\partial \phi(r-k)}{\partial t} = 2^j \dot{\phi}(r-k) \tag{11}$$

Substitute equation (11) into the state space equation (3):

$$\dot{z}(t) = 2^j \sum_k \alpha_k \dot{\phi}(r-k) = \dot{Z}(r) \tag{12}$$

## 4. Algorithm for instantaneous frequencies identification

### 4. 1 Transforming the state equations to linear algebraic equations

Substituting equations (10), (11) and (12) to state equations (3) and (4), we have:

$$\begin{cases} 2^j \sum_k \alpha_k \dot{\phi}(r-k) = A(\frac{r}{2^j})\sum_k \alpha_k \phi(r-k) + B(\frac{r}{2^j})\sum_k \beta_k \phi(r-k) \\ \sum_k \gamma_k \phi(r-k) = C(\frac{r}{2^j})\sum_k \alpha_k \phi(r-k) + D(\frac{r}{2^j})\sum_k \beta_k \phi(r-k) \end{cases} \tag{13}$$

Here let $\beta_k = \tilde{\beta}_k 2^{j/2}$ , $\gamma_k = \tilde{\gamma}_k 2^{j/2}$ , in which $\tilde{\beta}_k$ and $\tilde{\gamma}_k$ are the approximation coefficients of $u(t)$ and $y(t)$, having been multiplied by $\phi(r-l)$, and taking the inner product of both side of equation (13):

$$\begin{cases} 2^j \sum_k \alpha_k \int \dot{\phi}(r-k)\phi(r-l)dr = A(r/2^j)\sum_k \alpha_k \int \phi(r-k)\phi(r-l)dr \\ \qquad + B(r/2^j)\sum_k \beta_k \int \phi(r-k)\phi(r-l)dr \\ \sum_k \gamma_k \int \phi(r-k)\phi(r-l)dr = C(r/2^j)\sum_k \alpha_k \int \phi(r-k)\phi(r-l)dr \\ \qquad + D(r/2^j)\sum_k \beta_k \int \phi(r-k)\phi(r-l)dr \end{cases} \tag{14}$$

Using the translation orthogonality of the scaling functions $\int \phi(r-k)\phi(r-l)dr = \delta_{lk}$ , the equation (14) is rewritten as:

$$\begin{cases} 2^j \sum_k \alpha_k \Gamma_{l-k}^{(1)} - B_l \beta_l = A_l \alpha_l \\ \gamma_l - D_l \beta_l = C_l \alpha_l \end{cases} \tag{15}$$

in which $\Gamma_{l-k}^{(1)} = \int \dot{\phi}(r-k)\phi(r-l)dr$ are the connection coefficients. Without loss of the generality, it is assumed that the length of the signal $x(t)$ is $t_f \in Z$, hence we have $2^j t_f = n$. When the translation factors $k = [l-2N+2, l+2N-2]$ and the integers $l = 0,1,\cdots,n-1$, the connection coefficients $\Gamma_{l-k}^{(1)}$ are nonzero. If the systemic mass has been known, the system matrix $A(t)$ and the output influence matrix $C(t)$ can be identified by solving the linear equations.

### 4. 2 Free vibration case

When the excitation force term of the equation (1) is zero, the differential equations can be simplified. In sate equations, system matrix $A(t)$ and output influence matrix $C(t)$ are time-dependent. However, input matrix $B(t)$ and direct transmission matrix $D(t)$ do not exist any longer. According to the section 3, state vector $z(t)$ and the output $y(t)$ can be projected by using scaling functions of $dbN$ wavelet at resolution $j$, then doing variable substitution and doing first derivative to scaling function $\phi(\cdot)$. Following the procedures of section 4.1, introducing signals' Daubechies wavelet projection to the free vibration state equations:

$$\begin{cases} 2^j \sum_k \alpha_k \int \dot{\phi}(r-k)\phi(r-l)dr = A(\frac{r}{2^j}) \sum_k \alpha_k \int \phi(r-k)\phi(r-l)dr \\ \sum_k \beta_k \int \phi(r-k)\phi(r-l)dr = C(\frac{r}{2^j}) \sum_k \alpha_k \int \phi(r-k)\phi(r-l)dr \end{cases} \tag{16}$$

taking the inner product of both sides of equation (16), after having been multiplied by $\phi(r-l)$, using the orthogonality of the translates of the scaling functions, equation (16) is rewritten to:

$$\begin{cases} 2^j \sum_k \alpha_k \Gamma_{l-k}^{(1)} = A_l \alpha_l \\ \beta_l = C_l \alpha_l \end{cases} \tag{17}$$

where the connection coefficients $\Gamma_{l-k}^{(1)}$, translation factors $k$ and the integers $l$ all have the same definitions to the section 4.1.

Solving the linear equations above, the time-varying system matrix $A(t)$ and output influence matrix $C(t)$ can be determined.

### 4. 3 Procedures for instantaneous frequency identification

Taking the eigen decomposed to the systemic state matrix $A(t)$ at different time instant:

$$A(t) = \psi(t)\Lambda(t)\psi^{-1}(t) \tag{18}$$

where $\Lambda(t) = diag(\lambda_i(t), \lambda_i^*(t)\ ) \in C^{2p\times 2p}$ is a diagonal matrix, $\psi(t) \in C^{2p\times 2p}$ is a matrix of eigenvector, $\lambda_i(t), \lambda_i^*(t) = -\zeta_i(t) 2\pi\omega_i(t) \pm j2\pi\omega_i(t)\sqrt{1-\zeta_i^2(t)}$ , where $\omega_i(t)$ , $\zeta_i(t)$ are natural circular frequency and damping ration respectively, which can be defined as following, in which $i = 1, \cdots, p$ indexes the modal order:

$$\omega_i(t) = \frac{1}{2\pi}\sqrt{(\mathrm{Re}(\lambda_i(t)))^2 + (\mathrm{Im}(\lambda_i(t)))^2} \tag{19}$$

$$\zeta_t(t) = \frac{-\mathrm{Re}(\lambda_i(t))}{2\pi\omega_i(t)} \tag{20}$$

## 5. Numerical simulations

A 4-DOF LTV model (Fig. 1) is used for the case study.

The lumped mass coefficients are $M_1 = M_2 = M_3 = M_4 = 1\mathrm{kg}$.

The damping coefficients are $E_1 = E_4 = 0.6\mathrm{Ns/m}$, $E_2 = 0.65\mathrm{Ns/m}$, $E_3 = 0.7\mathrm{Ns/m}$.

The stiffness coefficients are $K_1 = K_2 = K_3 = K_4 = 40000\mathrm{N/s}$.

The free and forced vibration response sequences are calculated from numerical solution of the equation of motion using Newmark-beta method. The free vibration response is simulated with an initial displacement at the four lumped masses as $x_{1,2,3} = 0,\quad x_4 = 0.05m$ , while the initial velocity and acceleration are all zero. Two types of forced responses are calculated under multi harmonic excitation and random excitation with an all-zero initial condition. The multi harmonic excitation is $u(t) = 200 * (\sin 16\pi t, \sin 32\pi t, \sin 76\pi t, \sin 104\pi t)^T$ and the random excitation acting at four lumped masses, which is a normal distribution random value with mean zero and standard deviation fifty. In this example, we choose $db3$ Daubechies wavelet, and the resolution $j = 11$, so the sampling frequency of the time series data is $2^j = 2048$ Hz.

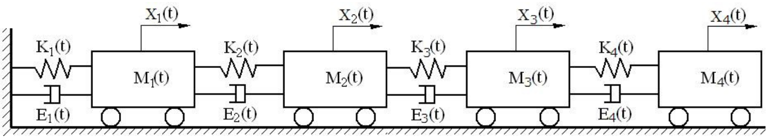


**Fig. 1.** 4-DOF lumped mass model

Different kinds of parameters with different kinds of time-varying cases (abrupt, smooth and periodic) are investigated. The instantaneous frequencies are identified using the proposed method from the free and forced responses (multi harmonic and random excitation).

**Case A. Mass coefficient and single stiffness coefficient are time varying.**

$$M_4(t) = 1 - 0.01t\ ;\ \ K_2(t) = \begin{cases} 40000, 0 \le t \le 0.375, \\ 32000, 0.375 < t \le 0.75, \\ 32000 + 4000\sin 1.33\pi t, 0.75 < t \le 2.25, \\ 32000 - 2000t, 2.25 < t \le 3. \end{cases}$$

The remaining parameters are invariant.

Figs. 2 and 3 provide the identified instantaneous frequency of the LTV system. Four frequencies can be tracked well using both free and forces responses. However, it is noted that there exists larger identification error at the time instance when the system parameters have an abrupt change. This indicates that the proposed algorithm has poor capability in tracking abrupt variations because of combining a number of time points during the calculation procedure of solving linear equations.

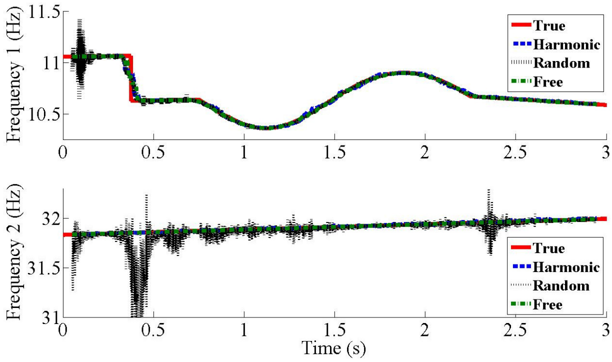


**Fig. 2.** Identified instantaneous frequencies (1st and 2nd order)

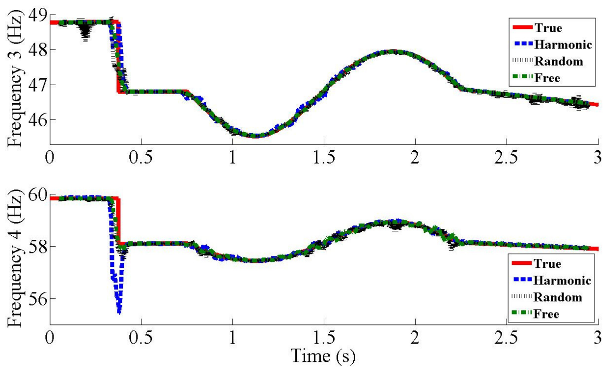


**Fig. 3.** Identified instantaneous frequencies (3rd and 4th order)

**Case B. Mass coefficient and multi stiffness coefficients are time varying.**

$$M_4(t) = 1 - 0.01t \; ; \quad K_2(t) = \begin{cases} 40000, 0 \le t \le 0.375, \\ 32000, 0.375 < t \le 0.75, \\ 32000 + 4000\sin 1.33\pi t, 0.75 < t \le 2.25, \\ 32000 - 2000t, 2.25 < t \le 3, \end{cases}$$

$$K_4(t) = \begin{cases} 40000, 0 \le t \le 0.375, \\ 28000, 0.375 < t \le 0.75, \\ 28000 + 4000\sin 1.33\pi t, 0.75 < t \le 2.25, \\ 28000 - 2000t, 2.25 < t \le 3. \end{cases}$$

The remaining parameters are invariant.

**Case C. Mass, damping and stiffness coefficients are time varying.**

$$M_4(t) = 1 - 0.01t\ ;\quad E_1(t) = 0.6 + 0.05t\ ;\quad K_2(t) = \begin{cases} 40000, 0 \le t \le 0.375, \\ 32000, 0.375 < t \le 0.75, \\ 32000 + 4000\sin 1.33\pi t, 0.75 < t \le 2.25, \\ 32000 - 2000t, 2.25 < t \le 3. \end{cases}$$

The remaining parameters are invariant.

A comparison is shown in Figs. 4 -7 between the true value and the identified value. Results indicate that the proposed algorithm has a good capability of tracking the time-varying frequency of the LTV system. The estimated results from free vibration responses are more accurate than those from the forced responses during the whole identification time period.

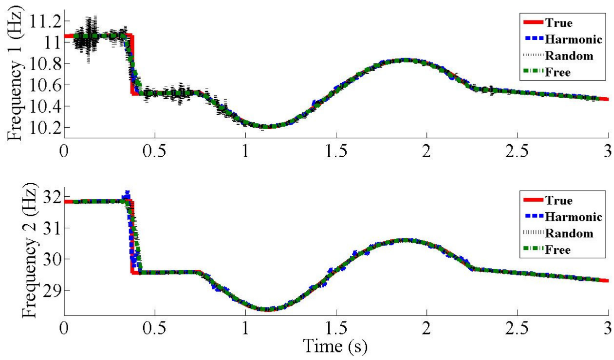


**Fig. 4.** Identified instantaneous frequencies (1st and 2nd order)

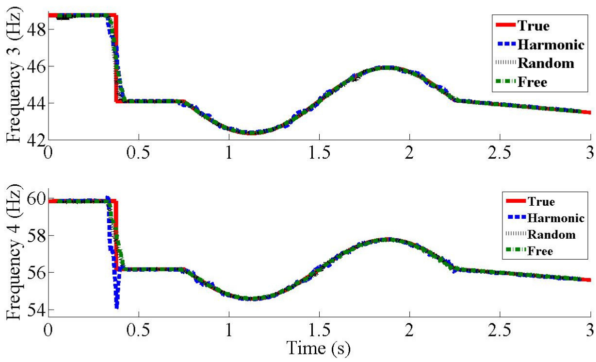


**Fig. 5.** Identified instantaneous frequencies (3rd and 4th order)

## 6. Conclusions

We presented a new technique to identify the instantaneous frequency of the LTV system from free or forced responses data using wavelet and state space methods. The proposed algorithm not only successfully avoids computing the connection coefficient between wavelet scaling functions and the second order derivative of wavelet scaling functions, but also significantly reduces the computation time in comparison to many subspace algorithms and time series recursive algorithms. A 4-DOF discrete model with three kinds of cases (abrupt, smooth and periodic) was investigated. Numerical results demonstrate that the proposed algorithm has a good capability for tracking the changes of instantaneous frequency of the LTV system.

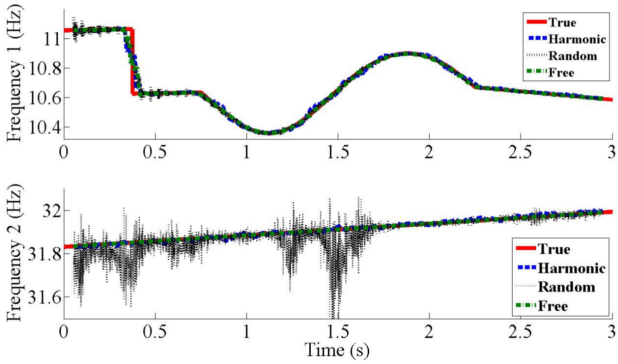


**Fig. 6.** Identified instantaneous frequencies (1st and 2nd order)

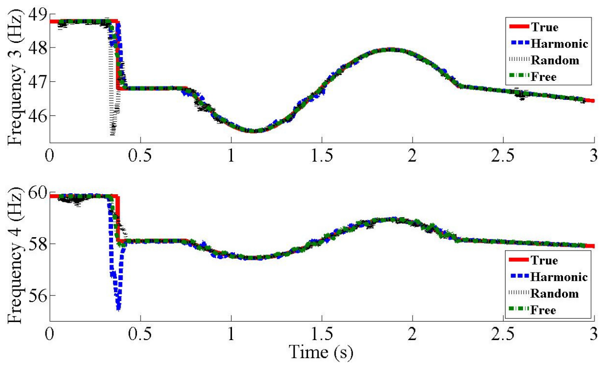


**Fig. 7.** Identified instantaneous frequencies (3rd and 4th order)

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