775. Introduction to synthesis of the torsional vibrating discrete-continuous mechatronic systems by means of the hypergraphs and structural numbers method

Andrzej Buchacz
Silesian University of Technology, 44-100 Gliwice, 18 A Konarskiego st., Poland
E-mail: *andrzej.buchacz@polsl.pl*

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Abstract. In this paper the modeling by a different category graphs and an analysis of vibrating clamped – free mechatronic system by the approximate method called Galerkin's method has been presented. The frequency - modal analysis and assignment of amplitude -frequency characteristics of the mechatronic system is considered. The aim was to nominate the relevance or irrelevance between the characteristics obtained by exact - only for shaft - and approximate method. Such formulation especially concerns the relevance of the natural frequencies - poles of characteristics both of mechanical subsystem and the discrete – continuous clamped – free vibrating mechatronic system. This approach is a fact, that approximate solutions fulfill all conditions for vibrating mechanical and/or mechatronic systems and can be an introduction to synthesis of these systems modeled by different category graphs. Using the hypergraph methods of modeling and synthesis methods of torsionally vibrating bars to the synthesis of discrete-continuous mechatronic systems is originality of such formulation of problems.

Keywords: vibrating continuous-discrete mechatronic system, Galerkin's method, graphs and structural numbers, modeling, synthesis.

Introduction

In the research Centre in Gliwice the problems of analysis of vibrating beam systems, discrete and discrete-continuous mechanical systems by means of the structural numbers methods modeled by the graphs and hypergraphs, have been investigated (e.g. [3-6]). The synthesis[1] of a selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems has been dealt in [3-5].

The approximate method of analysis, that means the orthogonalization method [10] and Galerkin's method [11], has been used to obtain the frequency-modal characteristics. The continuous-discrete torsional and transverse vibrating mechatronic systems were considered in [6, 7]. Transformations of hypergraps of flexibly vibrating beams were presented in [9]. To compare the obtained dynamical characteristics – dynamical flexibilities only for mechanical torsional vibrating bar and transverse vibrating beam being a parts of complex mechatronic systems, an exact method and the Galerkin's method were used [8,11]. Such formulation can be an introduction to synthesis of vibrating mechatronic systems which will lead to generating the vibrations with required parameters.

Dynamical Characteristic of Torsional Vibrating Mechatronic System

Therefore it becomes necessary to search the new solutions, having on aim the reduction of movable elements as well as compiled and long kinematic chains. From here in last years it is clear that there is a huge development on the market, especially in field of new technologies

[1] The methods of analysis and synthesis of electrical systems were presented in monograph [1].

basing on phenomenon of piezoelectricity, electro - and the magnetostriction (e.g. [12]). The piezoelectric elements are used to eliminate the oscillation [13].

Considered vibrating systems are presented in Fig. 1.

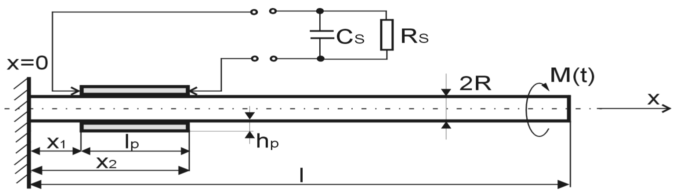

Fig. 1. The torsional vibrating mechatronic systems with a mechanical excitation

The set of equations of the mechanical and electric part of the system is following:

$$\begin{cases} \ddot{\varphi} - a^2\varphi - bU\left[\delta(x-x_1) - \delta(x-x_2)\right] = cM\delta(x-l), \\ \dot{U} + \alpha_1 U + \alpha_2\dot{\varphi}(l_p, t) = 0, \end{cases} \tag{1}$$

where: $a = \sqrt{\dfrac{G}{\rho}}$, $b = \dfrac{-\lambda^*}{I_o l\rho}$, $c = \dfrac{1}{I_o l\rho}$, $\alpha_1 = \dfrac{1}{R_s C_p}$, $\alpha_2 = \dfrac{2\pi R^2 h_p d_{15} G_p}{l_p C_p}$, $\delta(\cdot)$ - Dirac's function, G – the Kirchoff's modulus, ρ – the mass density, I_0 – the polar moment of inertia for a shaft, l – length of shaft, $\lambda^* = \dfrac{2}{3}\pi G_p\left[(R+h_p)^3 - R^3\right]\dfrac{d_{15}}{l_p}$, G_p – the Kirchoff's modulus of piezoelectric, $C_p = 2\pi R h_p \dfrac{e_1}{l_p}\left(1 - \dfrac{2d_{15}G_p}{e_1}\right) + C_x$ (C_x – an additional capacity in a short circuit system).

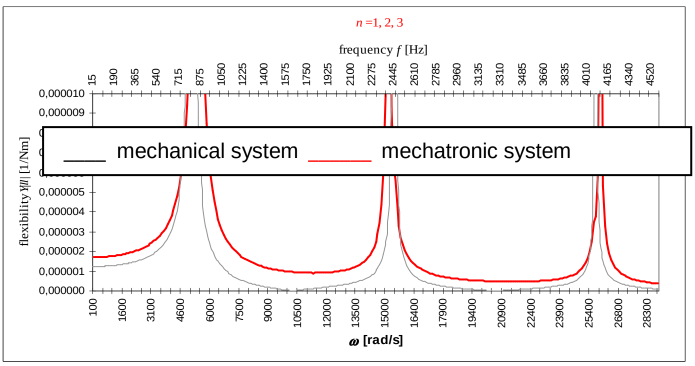

Fig. 2. Transient of the sum for n = 1, 2, 3 vibration mode

Solving a set of equation (1) after transformations [7-11] the dynamic flexibility takes a form:

$$Y = \sum_{n=1}^{\infty} Y_{xl}^{(n)} = \frac{c\delta(x-l)\left(\omega + \dfrac{\alpha_1}{e^{i\frac{\pi}{2}}}\right)}{\sin kx\left[\alpha^2\left(\dfrac{\pi}{2l}\right) - \omega^2\right]\left(\omega + \dfrac{\alpha_1}{e^{i\frac{\pi}{2}}}\right) - \dfrac{b}{\left(e^{i\frac{\pi}{2}}\right)^2}\delta(x)\alpha_2 \sin kl_p} \tag{2}$$

The transient of an absolute value of the flexibility in a considered range of frequency and flexibility for a three first vibration modes (2) - after further formal transformations and after putting of the numerical values of parameters and when *x* = *l*, that is $\alpha_Y = |Y_{ll}|$ - is presented in Fig. 2.

Transformations of Characteristics of Torsional Vibrating Subsystems of Mechatronic Systems

The problem consisted in modeling of torsional vibrating multiple-segment with mechanical bar systems as subsystems of mechatronic systems in the form of models with uniformly distributed parameters and a constant section in the segment.

In the modeling of the considered class of systems, the dependence between the amplitudes of generalized forces ${}_2s_k \in {}_2S$ and generalized displacements ${}_1s_i \in {}_1S$ can by described by dynamical flexibility Y_{ik} [3].

A characteristic – a dynamical flexibility of mechanical subsystem of mechatronic system is given in form:

$$Y(s) = \frac{\sum_{i=0}^{k} c_i th^j \Gamma s}{s \sum_{j=0}^{l} d_j th^j \Gamma s}. \tag{3}$$

After transformations of the flexibility (3) into mobility [3, 10, 11] and Wyndrum's one the mobility function has been obtained as:

$$V(r) = \frac{\sum_{i=0}^{k} c_i r^i}{\sum_{j=0}^{l} d_j r^j} \tag{4}$$

where: $c_i, c_{i-1}, \dots, c_0, d_j, d_{j-1}, \dots, d_0$ are any real numbers, $\Gamma = \sqrt{\frac{\rho}{G}} L = \sqrt{\frac{\rho^{(i)}}{G^{(i)}}} L^{(i)}$, ρ – mass density, $L=L^{(i)}$ – a length of basic element [3], $s = j\omega$, $j = \sqrt{-1}$, *i*, *j*, *k*, *l* – natural numbers, $k - l = 1$.

Modeling the Considered Subsystems of Mechatronic Systems by Means of the Different Category Graphs

A review of essential concepts of the graph theory, to fix a meaning of necessary terms and symbols have been presented before modeling the torsional vibrating continuous bar systems as subsystems of mechatronic systems and problems connected with it. Weighted hypergraphs (called in this paper also the weighted block graphs or weighted graphs of category *k*) have been applied to modeling of the considered systems. Definitions of graphs, as mathematical objects, have been presented on the basis of the literature. The bibliography of this subject is a very extensive and regards the theory as well as its applications (see [1-3]).

The hypergraph is called a couple:

$$ {}^{k}X = \left({}_{1}X,\ {}_{2}^{k}X \right), \tag{5} $$

where: ${}_{1}X = \left\{ {}_{1}x_0,\ {}_{1}x_1,\ {}_{1}x_2,\ \ldots,\ {}_{1}x_n \right\}$ – finite set of vertices, ${}_{2}^{k}X = \left({}_{2}^{k}X^{(i)} / i \in \mathrm{N} \right)$, ($k$ = 2, 3, ... $\in$ N) is a family of subsets of set ${}_{1}X$; the family ${}_{2}^{k}X$ is called a hypergraph over ${}_{1}X$ as well, and ${}_{2}^{k}X = \left\{ {}_{2}^{k}X^{(1)},\ {}_{2}^{k}X^{(2)},\ \ldots,\ {}_{2}^{k}X^{(m)} \right\}$ is a set of edges [2], called hyperedges or blocks, if:

$$ \begin{cases} {}_{2}^{k}X \neq \varnothing \ (i \in \mathrm{I}), \\ \bigcup_{i \in \mathrm{I}} {}_{2}^{k}X^{(i)} = {}_{2}X. \end{cases} \tag{6} $$

In this paper hypergraphs – graphs of category k – ${}^{k}X$ (k = 2, 3) are used, which will be clearly mentioned each time, as well as graphs X, called also graphs of the first category – ${}^{1}X$ (see [3]). The basic notions are shown in literature (e.g. [3, 10, 11]).

Resuming a set of generalized displacements of a torsional vibrating subsystem of mechatronic system can be formulated as follows: ${}_{1}S^{(i)} = \left\{ {}_{1}s_0^{(i)},\ {}_{1}s_1^{(1)},\ {}_{1}s_2^{(2)} \right\}$.

Determining one-to-one transformation, that:

$$ f\!:\ {}_{1}S^{(i)} \to {}_{1}X^{(i)},\ {}_{1}s_j^{(i)} \in {}_{1}S^{(i)},\ {}_{1}x_j^{(i)} \in {}_{1}X^{(i)},\ \ j = 0,1,2, \tag{7} $$

the hypergraph of bar – subsystem of mechatronic system is obtained:

$$ {}^{2}\underset{f}{X}{}^{(i)} = \left[{}^{2}X^{(i)}, f \right], \tag{8} $$

where: ${}^{2}X^{(i)} = \left({}_{1}X^{(i)},\ {}_{2}^{k}X^{(i)} \right)$, ${}_{1}X^{(i)} = \left\{ {}_{1}x_0^{(i)},\ {}_{1}x_1^{(i)},\ {}_{1}x_2^{(i)} \right\}$, ${}_{2}^{k}X^{(i)}$ – one-element family – three-element subset of vertices ${}_{1}X^{(i)}$.

Investigating i-th segment in a n-segment mechanical bar as a subsystem of mechatronic system with a constant section, the hypergraph model ${}^{k}\underset{f}{X}{}^{(i)}$ is introduced.

Introducing f_1 that assigns the generalized displacements to vertices ${}_{1}x_j^{(i)}$ of hypergraph ${}^{2}\underset{f}{X}{}^{(i)}$ as:

$$ f_1\left({}_{1}x_j^{(i)} \right) = \left| {}_{1}s_j^{(i)} \right|,\ (j = 0,1,2), \tag{9} $$

weighted hypergraph is obtained:

$$ {}^{2}\underset{1}{X}{}^{(i)} = \left[{}^{2}\underset{f}{X}{}^{(i)}, f_1 \right]. \tag{10} $$

On the basis of this assumption, a geometrical representation of mapping (8-10) has been shown in [3].

The Synthesis of the Mechanical and/or Mechatronic Systems Represented by Different Category Graphs

Methods which were applied used in order to synthesize the dynamical characteristic of the torsional vibrating mechanical systems are presented in this paper. They may be applied to synthesis the mechatronic system with cascade structure as well [3].

Algorithm of Synthesis of Mechanical Subsystem by the Recurrent Cascade Method as Necessary Condition of Synthesis of Mechatronic System

The recurrent formula[2] for synthesis subsystem of mechatronic system takes form of:

$$V^{(i+1)}(p)=\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}\frac{V^{(i)}(p)+\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}p}{\left(\frac{l}{GJ}\right)^{(i)}\frac{1}{\gamma}-V^{(i)}(p)p}, \tag{11}$$

where: $V^{(i+1)}(p)$ is the mobility of the transforming vibration system after its basic element with mobility $V^{(i)}(p)$ is removed, J – polar inertial moment of the bar cross section.

Equation (11) makes possible a determination the next (i+1) (i = 1, 2, ... ,n) dynamical characteristic of the transforming vibration system synthesized in this way.

To carry out the synthesis of the mobility $V(p)$ the form of (11) or its inversions $U(p)$ (comp. with [3]) by the cascade method it is necessary to:

1^0 Assume in (11) that $V(p)=V^{(1)}(p)$.

2^0 Determine values $(GJ)^{(i)}=\dfrac{1}{\beta V^{(i)}(1)}$, $(\rho J)^{(i)}=\dfrac{(GJ)^{(i)}}{G}\rho$ assuming p = 1 and i = 1, $\beta=\sqrt{\dfrac{\rho}{G}}$.

3^0 Determine $V^{(2)}(p)$ of other part of system containing segments from i = 2 to n, from Richards' theorem [3].

4^0 Divide the numerator and denominator of mobility $V^{(2)}(p)$ by (p^2-1); this is a condition of the physical realization of calculated mobility $V^{(2)}(p)$.

5^0 Repeat step 2, assuming i = 2.

6^0 Carry out step 3^0 in order to calculate $V^{(3)}(p)$.

7^0 Check step 4^0 by dividing the numerator and denominator of $V^{(3)}(p)$ by (p^2-1).

8^0 Repeat steps 2^0, 3^0, 4^0, ... successively to determine formulas $V^{(4)}(p)$, $V^{(5)}(p)$, ... , $V^{(n)}(p)$.

The algorithm described above will be continued until type p or $\dfrac{1}{p}$ of mobility $V^{(n)}(p)$ is achieved - multiplied by real constant H - and it is not possible to carry out step 3^0 after step 2^0 in order to determine $(GJ)^{(n)}$ and $(\rho J)^{(n)}$. This is the end of the synthesizing process.

Last Remarks

Applied method and received results can make up the introduction to the synthesis of considered class systems – torsional vibrating mechatronic ones with constant changeable cross-section. The problems will be presented in future works.

[2] The formula was obtained for longitudinally vibrating bar systems in [3].

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References

[1] **Bellert S., Woźniacki H.** The Analysis and Synthesis of Electrical Systems by Means of the Method of Structural Numbers. WNT, Warszawa, 1968, (in Polish).
[2] **Berge C.** Graphs and Hypergraphs. American Elsevier Publishing Co., Inc., New York / North Holland Publishing Co., Amsterdam – London, 1973.
[3] **Buchacz A.** The Synthesis of Vibrating Bar-Systems Represented by Graph and Structural Numbers. Scientific Letters of Silesian University of Technology, Mechanics, z. 104, Silesian University of Technology Press, 1991, (in Polish).
[4] **Buchacz A., Dymarek A., Dzitkowski T.** Design and Examining of Sensitivity of Continuous and Discrete-Continuous Mechanical Systems with Required Frequency Spectrum Represented by Graphs and Structural Numbers. Monograph No. 88, Silesian University of Technology Press, Gliwice, 2005, (in Polish).
[5] **Buchacz A.** The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs. Journal of Materials Processing Technology, Vol. 164-165, Complete Elsevier, 2005, p. 1277-1280.
[6] **Buchacz A.** Modeling, synthesis, modification, sensitivity and analysis of mechanic and mechatronic systems. Journal of Achievements in Materials and Manufacturing Engineering, International OCOSCO World Press, Vol. 24, Issue 1, September 2007, p. 198-207.
[7] **Buchacz A.** Dynamical flexibility of discrete-continuous vibrating mechatronic system. Journal of Achievements in Materials and Manufacturing Engineering, International OCOSCO World Press, Vol. 28, Issue 2, January 2008, p. 159-166.
[8] **Buchacz A.** Characteristics of discrete-continuous flexibly vibrating mechatronic system. Journal of Achievements in Materials and Manufacturing Engineering, International OCOSCO World Press, Vol. 28, Issue 1, May 2008, p. 43-46.
[9] **Buchacz A.** Transformations of hypergraphs of beams as models in synthesis of flexibly vibrating continuous mechanical systems. Herald of the Crimean Academy of Sciences – Sevastopol Department, Release 1, Sevastopol, 2010, p. 35-38.
[10] **Buchacz A.** Orthogonalization method of analysis of mechanical and mechatronic systems. Les Problèmes Du Technoshère Et De La Formation Des Cadres D'Ingénieurs Contemporains, Recueil des exposés des participants du III Séminaire International Scientifique et Méthodique, Novembre 2009, à Sousse (Tunisie), Donetsk, 2009, p. 85-88.
[11] **Buchacz A.** Comparision of characteristics of subsystems and systems as introduction to synthesis of torsionally vibrating mechatronic systems. Les Problèmes Du Technoshère Et De La Formation Des Cadres D'Ingénieurs Contemporains, Recueil des exposés des participants du IV Séminaire International Scientifique et Méthodique, Novembre 2010, à Hammamet (Tunisie), Donetsk, 2010, p. 10-13.
[12] **Callahan J., Baruh H.** Vibration monitoring of cylindrical shells using piezoelectric sensors. Finite Elements in Analysis and Design, Vol. 23, 1996, p. 303-318.
[13] **Kurnik W.** Damping of mechanical vibrations utilizing shunted piezoelements. Machine Dynamics Problems, Vol. 28, No. 4, 2004, p. 15-26.