

# 764. Approach to modeling of thermal airflow dynamics

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(Received 15 January 2012; accepted 14 February 2012)

**Abstract.** This work considers the modeling of thermal airflow. The dynamic airflow is split on flat circular elements. The extrapolation method, which is referred to as adaptive extrapolation, is proposed. The extrapolation which is influenced by stochastically changing settings is the key for adaptive extrapolation. The change of these parameters is determined experimentally during real flight or using prediction techniques. This paper presents modeling with preset parameters. The thermal airflow is modeled by taking into account its inclination towards wind and the change of diameter in accordance with linear and nonlinear laws.

**Keywords:** thermal airflow, thermal modeling, finite elements, circular finite elements, interpolation, adaptive extrapolation, delay time.

## Introduction

Currently the progress in electronics has exceeded the most optimistic expectations of last century's scientists and intelligent autonomous systems which manage dynamic objects are being developed. Nowadays light autonomous aircrafts, which can perform flights according to preset navigational settings, are becoming more and more popular [1], [2], [3]. Rising air flow or thermal flow is a viable, external, renewable energy source useful for autonomous aircrafts [1], [4]. The usage of this power source during autonomous aircraft flight can significantly increase the range or flight time and safe inner energy reserves [5], [6].

Thermal airflows allow us to use renewable energy sources during the flight of the autonomous aircraft, which has a computing system based on simple artificial intelligence [7]. However, a large volume of computing operations requires computing systems with high performance. Such systems consume a lot of energy. The solution is to use calculation systems with low power consumption. The rapid development of computing techniques focuses on reducing energy consumption for computational operations [7].

The practical usage of the thermal airflow for autonomous flight of aircraft is inseparable from the simulation of rising air flow structure. Such model would allow to use the energy of the thermal air flow more efficiently and would create further flight tactics and strategy by taking into account the specific meteorological conditions.

In this work the model of thermal flow is investigated and the possibility to reduce resources used by the computing system is taken into account. The main goal is to provide the information on vertical velocity on a space point of interest to the autonomous flight control system of the thermal air flow for further flight path planning. The work presents the model of thermal airflow, which is suitable for computing systems with various technical capacity and has huge development potential. In this research a thermal air flow model by using limited computing resources has been created.

## Methods

Currently the following thermal air flow models are used: Navier – Stokes equations, finite element techniques and various numerical methods which simplify calculation of sophisticated dynamics of fluids and gases [9], [10].

Aerodynamic quality factor value of modern aircrafts often reaches up to 40 units. For example thrust needed for aircraft, the weight of which is 10 kg and the coefficient of aerodynamic quality is 30 units, horizontal flight equals 0.33 kg.

An electric motor, which consumes the amount of energy of tens of watts per hour, can produce the thrust of 0.33 kg. Modern high performance computing systems have the same power consumption. For this reason it is appropriate to reduce power consumption of computing systems in order to increase flight time of light autonomous aircraft.

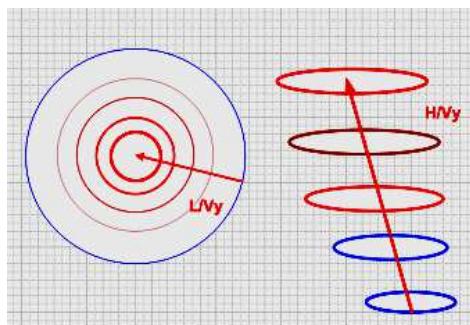
Since the simulation is performed by using a computing system with constant computing power, the simulation time of all thermal flow discrete points in space should depend only on the number of these points. All points must be modeled by using the same algorithm with a constant amount of instructions. Then the simulation time of set of all points  $\tau(Y)_i$ , which is executed during one simulation cycle – iteration, which matches to a moment,  $t_i$  can be expressed as the following dependence:

$$\tau(Y)_i = \sum_{n=1}^N \tau_{ni}(y) + \tau_{sk} + \tau_{const} \quad (1)$$

where  $N$  - the number of modeled points;  $\tau_{ni}(y)$  - calculation time of one point, which has the order index  $n$  of the  $N$  number of points during the iteration time  $i$  moment  $t_i$ ;  $\tau_{const}$  - constant delay time required for the execution of operating system instructions;  $\tau_{sk}$  - constant counting time used for other functions of the simulation algorithm.

The thermal flow structure is influenced by two important meteorological indexes. The first is referred to as CAPE (Convective Available Potential Energy). This index of convective instability indicates the amount of thermal activity of a given date and is measured in [J/Kg] or [ $^{\circ}\text{C}\cdot\text{km}$ ], [11]. Another index is the SRH (Storm – Relative Helicity). SRH units are [ $\text{m}^2/\text{s}^2$ ]. This index affects the strength of thermal airflow in cumulus clouds, also it affects thermal flow inclination towards wind direction [12]. However these indexes are relevant to a relatively large area of meteorological phenomena covering hundreds or thousands of square kilometers [12]. In this work the thermal flow is described as “meso – scale” meteorological phenomenon rarely reaching the horizontal width of one kilometer. The most common size for it is tens of meters, sometimes hundreds of meters [13].

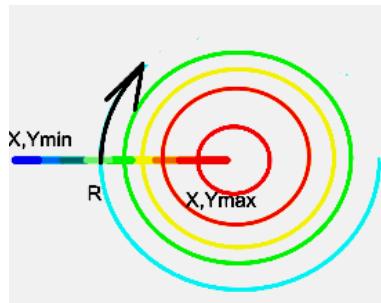
This work deals with the thermal flow simulation carried out with the following simplifications: the thermal airflow is regarded as a complete object which does not move in space ( $v_x = 0$ ,  $v_y = 0$ ,  $v_z \neq 0$ ) and is analyzed as a field of gradient parameters, the shape is constant and is set at the beginning of the simulation, the energy potential of thermal flow depends on the characteristics of the thermal flow, but in this case there is no change in the flow, accordingly the flow is not dynamic. Probability of flow detection is not considered in this work. It is estimated that there is no velocity gradient of the thermal flow at the vertical plane (Fig. 1).



**Fig. 1.** The concept of thermal airflow

In this work 3D simulation of thermal airflow is performed using Cartesian coordinate system. The starting point in coordinate system is at the bottom-center of the thermal flow. The flow is symmetrical to a horizontal plane. Flow in the vertical plane can be tilted due to wind drift. This inclination may be linear or nonlinear, it depends on the model parameters. The vertical plane flow may have radius, which changes from linear and nonlinear laws. This change depends on preset parameters. In a horizontal plane the flow has a gradient of vertical velocity component  $L/V_Y$ . The flow is modeled as concentric circles (Fig. 2). It is proposed to refer to these circles as “orbitals”. Orbitals are composed of flat disks.

Orbitals are modeled sequentially one after another, by moving in discrete steps from the thermal flow center towards its boundaries (Fig. 2). Each point is modeled sequentially by moving it by a preset angle in the orbital circle line during every iteration.

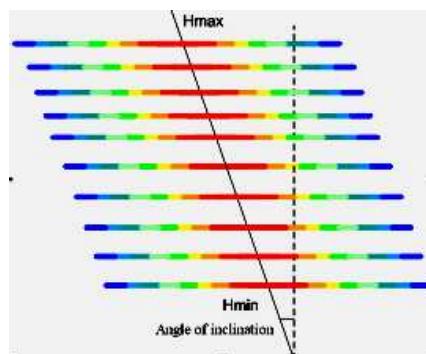


**Fig. 2.** Sequential modeling of orbitals

Each disk is a two-dimensional finite element of the flow. Whole two-dimensional elements create a three-dimensional thermal flow model. Each value of vertical velocity of thermal flow can be found by using the cubic interpolation method. You have to interpolate between the maximum and minimum values of flow velocity. Increased amount of two-dimensional finite elements N can increase modeling accuracy of thermal flow, but at the same time it increases the amount of computer resources used by a calculating system.

The most frequently used interpolation algorithms are: Shepard's interpolation; Radial Basis; Cosine Expansion; Polynomial; Triangulations; Mask Method. Mask Method provides flexible, high-quality calculations. Multistage methods such as Converging Average; Local B-spline; Global B-spline also are used in order to increase productivity of interpolation calculations [14], [15], [16].

Two-dimensional plate elements are modeled by the longitudinal flow axis by using extrapolation methods (Fig. 3).



**Fig. 3.** Extrapolation of plate elements

Extrapolation methods are similar to interpolation methods. This paper proposes to use the extrapolation method, which is similar to linear extrapolation, but this extrapolation is carried out in accordance with certain meteorological parameters of the surrounding area. The proposed term for this method is “Adaptive extrapolation”. Traditional interpolation methods are not practical for extrapolation of thermal flow finite element. Changes of thermal flow meteorological parameters along the vertical axis have complex stochastic rules [9], [16], [10].

Parameters which are used for modeling of form of thermal flow and for other meteorological characteristics can be set by collecting meteorological probes measurement data, or by using an aircraft as a meteorological probe. There are other ways to measure or predict meteorological conditions for a given location for a specific date as well, for example, by artificial neural networks [18].

In order to increase the accuracy of modeled data in particular point  $W(y, x, z)$ , two-dimensional and three-dimensional cubic interpolation methods can be used.

In this work a two-dimensional cubic interpolation of one of the thermal flow flat elements 90° sector was performed. One-dimensional cubic splines on  $X$  and  $Y$  axis for two-dimensional cubic interpolation were determined. Another step was the differentiation of these splines. In the results gradient of interpolation function and second-order derivatives in the appropriate spots were obtained. Thermal flow is modeled by using C++ default Math.h library. Thermal visualization is created by using an OpenGL graphical driver.

## Results

As mentioned before, the flat circular element of the flow consist of concentric rings also known as orbitals. Each orbital is composed of discrete points, which have values of thermal flow vertical velocity component. The center of flow flat bottom element is in the center of the axis of coordinates. Orbital coordinates have been determined by the following equations:

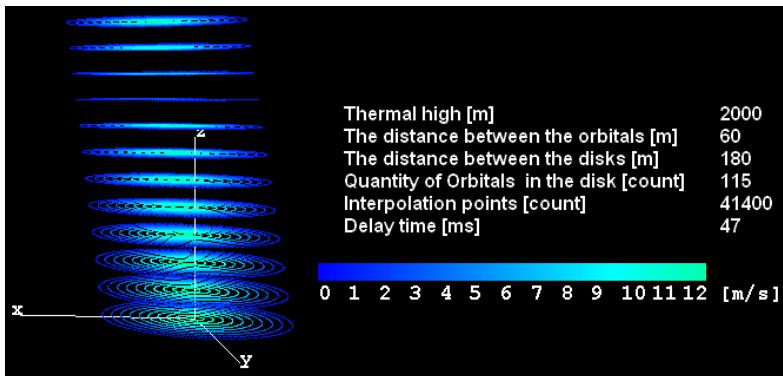
$$\begin{aligned} w_{ij}(x) &= R_{term} \cdot \sin\left(\theta \cdot \frac{Pi}{180}\right) + \\ &+ H_j \cdot \tan\left(\lambda \cdot \frac{Pi}{180}\right) \cdot \cos\left(\beta_V \cdot \frac{Pi}{180}\right) \\ w_{ij}(y) &= R_{term} \cdot \cos\left(\theta \cdot \frac{Pi}{180}\right) + \\ &+ H_j \cdot \tan\left(\lambda \cdot \frac{Pi}{180}\right) \cdot \sin\left(\beta_V \cdot \frac{Pi}{180}\right) \\ w_{ij}(z) &= H_j \end{aligned} \quad (2)$$

where  $R_{term}$  is the diameter [m] of thermal airflow,  $H_j$  - thermal flow height [m] at current iteration,  $\theta$  - iteration angle in the horizontal plane [degrees],  $\lambda$  - thermal flow inclination angle [degrees],  $\beta_V$  - angle of wind direction [degrees],  $i$  - iteration number within limits of a single disk,  $j$  - disks order number.

Flat flow elements have been modeled within a certain distance in a vertical plane. This distance depends on discrete height values  $H_D$  (Fig. 4):

$$H_i = H_{i-1} + H_D \quad (3)$$

where  $H_i$  is the value of height of the current modeled disk,  $H_D$  - variable height discretion.



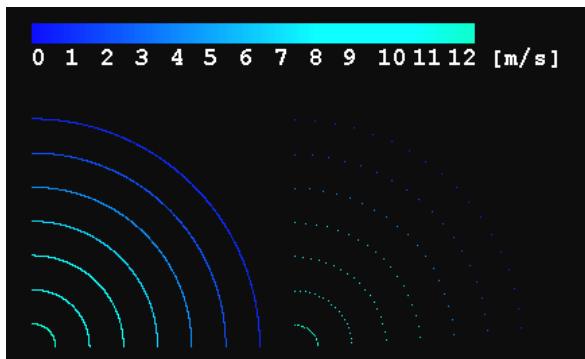
**Fig. 4.** Thermal airflow model with a constant pitch angle and width

Iteration angle  $\theta$  was raised in discrete steps  $\theta_D$  from zero to  $360^\circ$ . Each of the next value of iteration angle  $\theta_i$  was higher by an angle  $\theta_D$  than  $\theta_{i-1}$  (4):

$$\theta_i = \theta_{i-1} + \theta_D \quad (4)$$

This way the field points of gradient parameters are modeled around the orbitals circumference. Angle  $\theta$  discretion influences the accuracy of modeling.

Fig. 5 shows  $90^\circ$  a sector of modeled thermal airflow flat element, when  $\theta_D = 0.1^\circ$  (in the left) and when  $\theta_D = 5^\circ$  (in the right) (Fig. 5).



**Fig. 5.** Angle  $\theta$  discretization influence on a modeled orbital

The next step was to model the inclination angle of the thermal airflow. Such changing inclination angle can be caused by the wind velocity gradient along the vertical axis. Modeling of inclination angle was implemented by setting the following inclination angle dependence on height:

$$\lambda_i = \lambda_{i-1} + \frac{\lambda_i \cdot H_i}{K} \quad (5)$$

where  $\lambda_i$  is the value of inclination angle during present iteration,  $\lambda_{i-1}$  - the value of inclination angle during past iteration,  $K$  - constant coefficient with set impact of height on value of inclination angle,  $H_i$  - the value of height during present iteration.

The model of thermal airflow with smoothly increasing inclination  $\lambda_i$  according to dependence (5) is provided in Fig. 6.

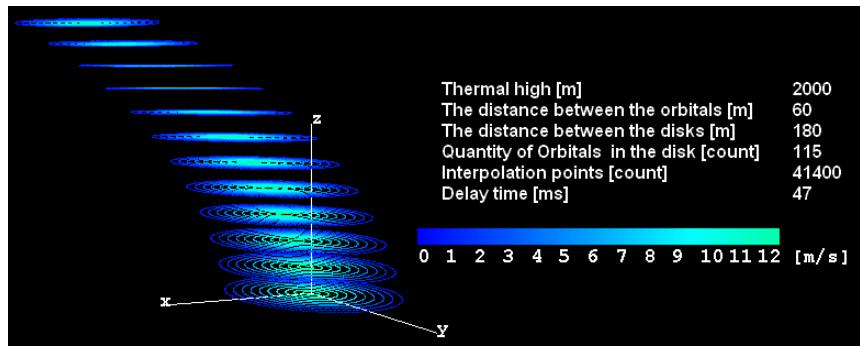


Fig. 6. Change of thermal airflow inclination

Thermal flow dependence on height has been modeled by using linear and nonlinear laws. According to the linear law, the diameter of the flat disk has been changed by using the value of height  $H_i$ :

$$R_i = R_0 + \frac{H_i}{K} \quad (6)$$

where  $R_i$  - radius of the flow at corresponding modeled height  $H_i$ ,  $R_0$  - initial radius of the flow at the beginning of coordinate system,  $K$  - constant coefficient.

The modeling result when the flow width increases linearly with height is presented in Fig. 7.

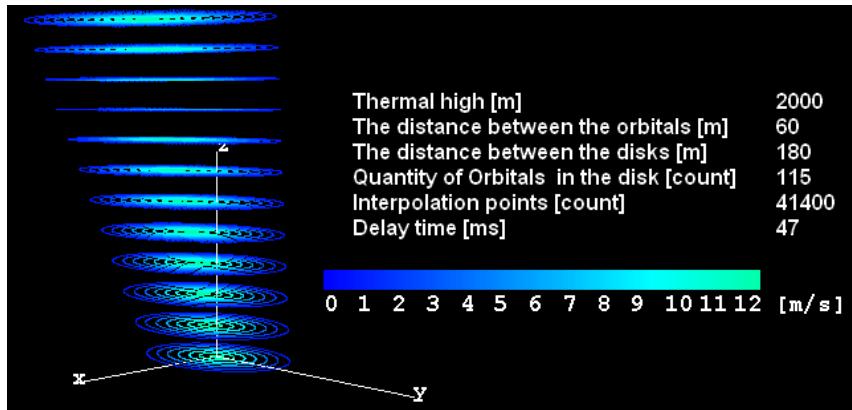


Fig. 7. Thermal airflow with linearly varying width

The law for non-linear flow width variation along the vertical axis (8) was also used. Modeling result when the radius of the flow changes non-linearly depending on height is given in Fig. 8:

$$R_i = R_0 + \frac{\cos\left(\frac{R_0 + H_i/L}{J}\right)}{K} \quad (7)$$

where  $R_0$  - initial value of thermal airflow radius at the beginning of coordinate system;  $R_i$  - value of thermal airflow radius;  $H_i$  - current modeling height;  $L, J, K$  - scaling coefficients (Fig. 8).

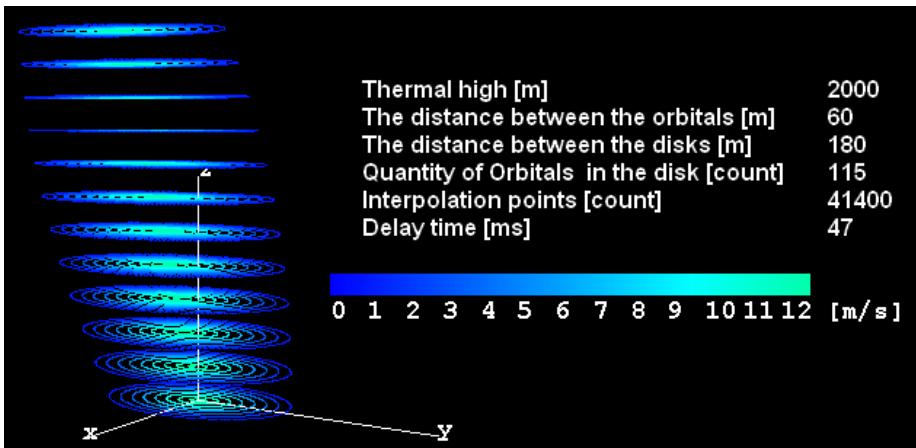


Fig. 8. Flow with non-linearly varying width

One 90° sector of thermal airflow flat element has been modeled by using a two dimensional rectangular parameter network. A two dimensional matrix which contains sixteen points was chosen. This matrix was the starting material in order to perform bicubic interpolation. Another 4×4 data matrix was set up for the interpolation:

$$A = \begin{bmatrix} 0 & 2 & 8 & 12 \\ 0 & 0 & 5 & 8 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The interpolation was carried out by using the recurrence method and interpolating every point with coordinates  $w(y, x)$  around the area between coordinates along the  $x$  axis (respectively -1...2 along to the  $y$  axis). Each point  $w(x, y)$  interpolants were found by calculating values of sixteen coefficients:

$$\begin{aligned} w(x, y) = & b_1 f(0,0) + b_2 f(0,1) + b_3 f(1,0) + \\ & b_4 f(1,1) + b_5 f(0,-1) + b_6 f(-1,0) + \\ & + b_7 f(1,-1) + b_8 f(-1,1) + b_9 f(0,2) + \\ & + b_{10} f(2,0) + b_{11} f(-1,-1) + b_{12} f(1,2) + \\ & + b_{13} f(2,1) + b_{14} f(-1,2) + b_{15} f(2,-1) + \\ & + b_{16} f(2,2) \end{aligned} \quad (8)$$

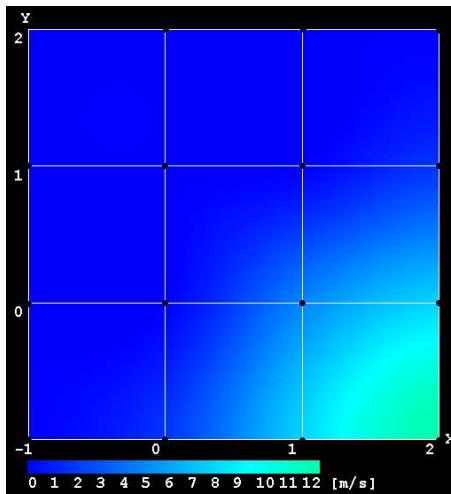
where  $B(i, j)$  are coefficients of nodes:

$$w(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j \quad (9)$$

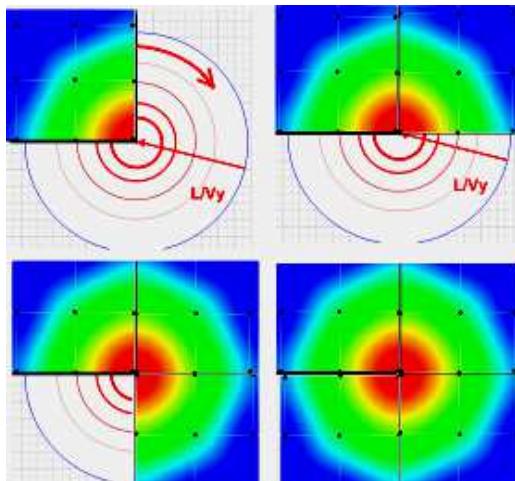
The result of two-dimensional cubic interpolation of matrix  $A[...][...]$  is shown in Fig. 9.

By having one sector of air flow cross-section and by adopting the simplification that the flow is symmetrical in horizontal plane, you can restore the entire image of airflow cross-section interpolation. This can be done by turning the image of flow interpolation around the maximum value point (Fig. 10). This turn of the interpolation function practically is in accordance with extrapolation methods.

Thermal airflow model uses computing system resources (1). The time of one modeling iteration, which depends on the amount of modeled points, has been studied.



**Fig. 9.** Dependence of time on the number of modeled points



**Fig. 10.** Thermal flow bicubic interpolation with extrapolation turn

**Table 1.** Time dependence on the amount of modeled points

Amount of points ×1000	Iteration time [ms]
684	407
272,16	180
149,04	109
90,72	78
64,8	63
46,8	55
39,6	47
28,8	40

The data in Table 1 presents linear dependence between the number of modeled points and the delay time of one iteration (Fig. 11). According to the range of autonomous light aircraft complete speeds, the time needed for modeling of tens of milliseconds duration flow is equivalent to a few meters distance which can be covered by aircraft within one iteration of flow modeling.

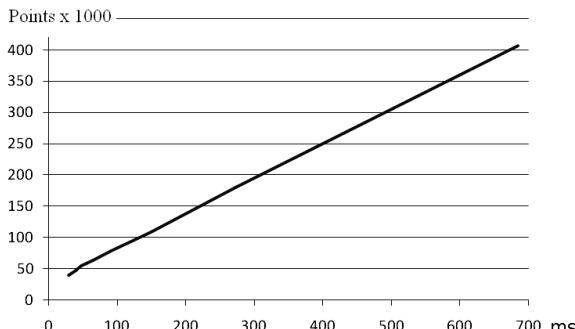


Fig. 11. Time dependency on the amount of modeled points

## Conclusions

In order to model dynamic thermal airflow the method of circular finite elements can be applied. Application of adaptive extrapolation method in order to model thermal airflow from flat panel finite elements, which are arranged in a three-dimension space, enables linear and non-linear model flows with varying width and with steady and non-steady inclination angle. The proposed methodology is suitable for thermal flow modeling with limited computing resources. This methodology allows us to reach the required detail level of thermal airflow model needed for flight vehicles and acceptable time expenses to create a single iteration of a model.

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