750. Damage detection based on the internal force or deformation variation

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(Received 15 December 2011; accepted 14 February 2012)

Abstract. The presence of damage in an intact structure leads to the change in internal force and deformation due to stiffness deterioration in the region of damage. This study proposes model-based damage detection methods by deriving the mathematical formulation to describe such changes. The force and deformation variations between the undamaged and damaged systems are derived by minimizing the variation in dynamic strain energy with respect to the internal force and deformation vectors, respectively. They are expressed by the product of a coefficient matrix and the external force vector, and the product of a coefficient matrix and the displacement vector, respectively. Taking singular value decomposition (SVD) on the coefficient matrices of rank-deficiency, this study identifies the damaged elements as belonging to the set of elements whose internal forces or deformations between two adjacent nodes of finite element model are not changed. The validity of the proposed methods is illustrated in a simple application.

Keywords: FRF, damage detection, singular value decomposition, internal force, minimization.

1. Introduction

Regular inspection and condition assessment of civil structures are necessary to allow early detection of any defect and to enable maintenance and repair works at the initial damage phase, so that the structural safety and reliability are guaranteed with a minimum of costs. Structural damage detection technique indicates the problem of how to locate and detect damage in a structure by using the observed changes in its dynamic characteristics.

There has been a lot of research endeavor for damage detection and assessment of structures by the dynamic approach using dynamic test data or the static approach using static test data. The spatially sampled field measured during the dynamic and static testing has been an active area of research for many years. The structural damage leads to the change in the static and dynamic characteristics of the initial system. It is interpreted that the change of physical properties provides basic information which can detect the damage.

Doebling et al. [1] presented a recent thorough review of various vibration-based damage identification methods. Doebling et al. [2] presented a method for identifying the local stiffness of a structure from vibration test data based on a projection of the experimentally measured flexibility matrix onto the strain energy distribution in local elements or regional superelements. Sheena et al. [3] presented an analytical method to assess the stiffness matrix by minimizing the difference between the actual and the analytical stiffness matrix subjected to the measured displacement constraints. Minimizing the difference between the applied and the internal forces, Sanayei and Scampoli [4] presented a finite element method for static parameter identification of structures by the systematic identification of plate-bending stiffness parameters for a one-third scale, reinforced-concrete pier-deck model. Sanayei and Onipede [5] provided an analytical method to identify the properties of structural elements from static test data such as a set of applied static forces and another set of measured displacements. Minimizing an index of discrepancy between the model and the measurements, Banan et al. [6, 7] proposed the mathematical formulations of two least-squares parameter estimators that evaluate element constitutive parameters of a finite-element model that corresponds to a real structural system from measured static response to a given set of loads. And they investigated the performance of the force-error estimator and the displacement-error estimator.

Hjelmstad et al. [8] proposed the mutual residual energy approach which can estimate complex linear structures. The method based on the principle of virtual work yields equations for estimating stiffness and mass parameters of linear structures by decomposing the system matrices. Hjelmstad and Shin [9] developed an analytical method based on a parameter estimation with an adaptive parameter grouping scheme to localize damage in a structural system for which the measured data are sparse. Cui et al. [10] developed a damage detection algorithm based on static displacement and strain. This method has a difficulty in requiring sufficient measurement information and load cases. Choi et al. [11] developed an elastic damage load theorem and an approach on the damage identification using static displacements. Chen et al. [12] presented a two-stage damage identification algorithm to use the change of measured static displacement curvature and grey system theory. Bakhtiari-Nejad et al. [13] presented a method to describe the change in the static displacement of certain degrees of freedom by minimizing the difference between the load vectors of damaged and undamaged structures. Wang et al. [14] proposed a two-stage identification algorithm for identifying the structural damages by employing the changes in natural frequencies and measured static displacements.

Pandey et al. [15] stated that once the displacement shapes of a damaged and of the corresponding undamaged structures are identified, the curvature can be obtained by a central difference approximation. The damage exists at the position to exhibit the abrupt increase of the curvature due to the deterioration of flexural rigidity.

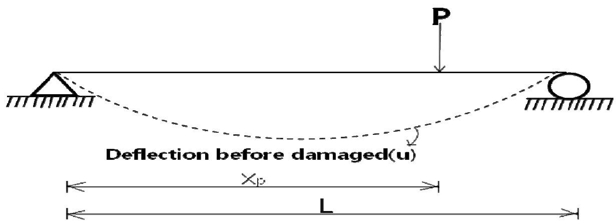
The change in flexibility or stiffness matrix is used for the damage identification. Starting from the theorem of minimum strain energy, Bernal [16] presented a damage localization method based on a change in measured flexibility. The technique identifies the elements of the structure that are damaged as belonging to the set of elements whose internal forces under the action of a set of load vectors are constant. Ratcliffe [17] proposed a method for locating structural damage using experimental vibration data without a priori knowledge about the undamaged structure. This method was derived under the assumption that the section of the intact structure is homogeneous and uniform without any defect.

Minimizing the variation in dynamic strain energy due to damage with respect to the internal force and displacement variation vectors, respectively, this study proposes damage detection methods to detect damage from the internal force or deformation variation. The local change of stiffness due to damage yields the rigid body motion by the damaged element only. It is shown that the damages located at the elements that are damaged as belonging to the set of elements whose internal forces or deformations are not changed. The method does not require the input data and can be easily utilized for locating the damage. The validity of the proposed method is illustrated in a simple application.

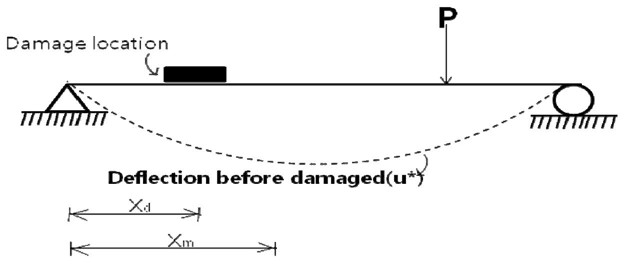
2. Formulation

Under the excessive increase in external forces or the action of unexpected loads, structures can be partially damaged and the stiffness is deteriorated due to damage. Thus, the variation in stiffness or flexibility matrix can be utilized as an index to detect the damage. The stiffness or flexibility variation is related to the internal force or displacement variations in the structure as the change of mechanical properties before and after the damage.

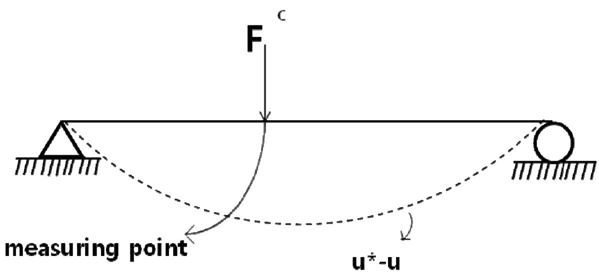
Considering a damaged beam structure under a set of external forces, the deflected curve of the damaged beam can be similarly described by the initial beam subjected to the external forces as well as unknown forces at several measurement positions. Figure 1(a) exhibits the deflected curve of an initial beam structure subjected to a concentrated load. As the load increases, the beam is likely to be damaged and Fig. 1(b) represents the deflected curve of the damaged beam.



(a)



(b)



(c)

Fig. 1. Damage detection approach; (a) deflected curve before damage, (b) deflected curve after damage, (c) additional displacement caused by the additional force on undamaged beam

The additional deflection of the beam due to the damage can be approximately estimated by the additional forces at measurement positions. Figure 1(c) represents the additional deflection $u^* - u$, where u^* and u indicate the deflection of the damaged and undamaged beam structures, respectively. The mathematical equations to represent the variation in the internal force and deformation are derived by minimizing the cost functions of force or displacement variations at measurement locations before and after the damage.

The dynamic behavior of a structure, which is assumed to be linear and approximately discretized for *n* degree-of-freedom, can be described by the equations of motion:

$$\mathbf{M}_a \ddot{\mathbf{u}} + \mathbf{C}_a \dot{\mathbf{u}} + \mathbf{K}_a \mathbf{u} = \hat{\mathbf{F}}(t) \tag{1}$$

where $\mathbf{M}_a$ and $\mathbf{K}_a$ denote the analytical $n \times n$ mass and stiffness matrices, $\mathbf{u} = [u_1 \quad u_2 \quad \cdots \quad u_n]^T$, and $\mathbf{C}_a \in R^{n \times n}$ is the damping matrix, and $\hat{\mathbf{F}}(t)$ is the $n \times 1$ load excitation vector. Without loss of generality, Rayleigh damping is adopted as:

$$\mathbf{C}_a = \alpha\mathbf{M}_a + \beta\mathbf{K}_a \tag{2}$$

where α and β are the two proportionality constants, which can be related to the damping ratios of the first and second natural modes. Assuming the system is lightly damped and considering the actual dynamic system due to damage, the dynamic equation of Eq. (1) becomes:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \hat{\mathbf{F}} \tag{3}$$

where $\mathbf{K}$ represents the actual stiffness matrix after the damage. And we assume that the mass is not changed as $\mathbf{M} = \mathbf{M}_a$.

Inserting $\mathbf{u} = \hat{\mathbf{U}}e^{j\Omega t}$ and $\hat{\mathbf{F}} = \mathbf{F}e^{j\Omega t}$ into Eq. (3) and expressing it as the form of frequency domain, it follows that:

$$\left(\mathbf{K} - \Omega^2\mathbf{M}\right)\hat{\mathbf{U}}(\Omega) = \mathbf{F}(\Omega) \tag{4}$$

where Ω denotes the frequency of the external force. The response of the original structure, described by $\hat{\mathbf{U}}(\Omega)$, to an external excitation, described by $\mathbf{F}(\Omega)$, is given by:

$$\mathbf{B}_0\hat{\mathbf{U}}(\Omega) = \mathbf{F}(\Omega) \tag{5}$$

where $\mathbf{B}_0(\Omega) = \left(\mathbf{K} - \Omega^2\mathbf{M}\right)$ is the impedance-type matrix or dynamic stiffness matrix of the original structure. It is shown that Eq. (5) of dynamic equation in the frequency domain takes the similar form as the static equilibrium equation. Modal transformation using the real eigenvalues and eigenvectors leads to the representation of the frequency response function (FRF) matrix for an excitation frequency Ω:

$$\mathbf{H}(\Omega) = \sum_{i=1}^{n} \frac{\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T}{\omega_i^2 - \Omega^2} \tag{6}$$

with $\hat{\mathbf{U}}(\Omega) = \mathbf{H}(\Omega)\mathbf{F}(\Omega)$ and $\mathbf{B}_0(\Omega) = \mathbf{H}^{-1}(\Omega)$. For the case of a displacement response at station *p* and a disturbing force at station *q* the numerical frequency response can be constructed as:

$$H_{p,q}(\Omega) = \sum_{i=1}^{n} \frac{\phi_{i,p}\phi_{i,q}}{\omega_i^2 - \Omega^2} \tag{7}$$

where $\phi_{i,p}$ denotes the *p*-th element of the vector ϕ_i.

The structural dynamic features can be changed by unexpected environmental change or damage of the system and should be determined based on the measurement modal data. Let us assume that from the modal displacements corresponding to a disturbing force at station *q*, their relative relation can be written as:

$$\mathbf{A}\,\mathbf{U}(\Omega) = \mathbf{0} \tag{8}$$

Equation (8) represents constraints to locally govern the dynamic responses, Ω denotes a specific frequency and $\mathbf{U}(\Omega)$ is the corresponding deformation shape. The dynamic stiffness matrix and FRF matrix in the frequency domain of Eq. (5) should be modified for satisfying the measured test data of Eq. (8).

We derive the internal force vector required for obtaining the response variation deviated from the initial response due to damage. Let us consider the n degree-of-freedom system described by $\hat{\mathbf{U}} = \left[\hat{U}_1 \quad \hat{U}_2 \quad \cdots \quad \hat{U}_n\right]^T$. The dynamic strain energy V_u, for the undamaged state is given by:

$$V_u = \frac{1}{2}\hat{\mathbf{U}}^T \mathbf{B}_0 \hat{\mathbf{U}} \tag{9}$$

where $\mathbf{B}_0$ denotes $n \times n$ positive-definite dynamic stiffness matrix. And the dynamic strain energy, V_d, for the damaged state is written as:

$$V_d = \frac{1}{2}\mathbf{U}^T \mathbf{B}\mathbf{U} \tag{10}$$

where $\mathbf{B}$ and $\mathbf{U}$ represent the dynamic stiffness matrix and displacement vector of the damaged system, respectively. Based on the minimum strain energy, a member of the set of admissible functions can be the undamaged displacement field and the substitution of the displacement field into Eq. (10) leads to the relation of:

$$\frac{1}{2}\hat{\mathbf{U}}^T \mathbf{B}\hat{\mathbf{U}} \geq \frac{1}{2}\hat{\mathbf{U}}^T \mathbf{B}_0 \hat{\mathbf{U}} \tag{11}$$

The dynamic stiffness of the damaged system can be written in terms of the initial dynamic stiffness:

$$\mathbf{B} = \boldsymbol{\lambda}\mathbf{B}_0 = \mathbf{B}_0 + \Delta\mathbf{B} \tag{12}$$

where $\boldsymbol{\lambda}$ is a coefficient matrix to represent the relation of initial and damaged dynamic stiffness matrices, and $\Delta\mathbf{B}$ denotes the dynamic stiffness variation matrix. The additional forces are defined as the constraint forces required additionally for obtaining the damaged responses at measurement positions of the initial system. The forces should explain the dynamic stiffness variation and can be written as:

$$\mathbf{F}^c = (\Delta\mathbf{B})\hat{\mathbf{U}} = \mathbf{C}_s\hat{\mathbf{F}} \tag{13}$$

where $\mathbf{C}_s$ is the coefficient matrix of the external force vector $\hat{\mathbf{F}}$ to describe the constraint force. The coefficient matrix is expressed as the function of its initial dynamic stiffness matrix $\mathbf{B}_0$. In the following, the control force $\mathbf{F}^c$ as well as the coefficient matrix is derived based on the variation in the dynamic strain energy.

The cost function is defined as the quadratic form of the variation in the dynamic strain energy between the undamaged and damaged states. The theory of minimum strain energy is interpreted that of all the admissible displacements that yield the correct displacement at the loaded coordinates, the actual displacement field that satisfies equilibrium minimizes the strain energy. The variation in the dynamic strain energy between the intact and damaged systems can be written as:

$$\delta U = \frac{1}{2}\left(\mathbf{U} - \hat{\mathbf{U}}\right)^T \mathbf{B}_0 \left(\mathbf{U} - \hat{\mathbf{U}}\right) = \frac{1}{2}\left(\mathbf{F}^c\right)^T \mathbf{B}_0^{-1}\mathbf{F}^c \tag{14}$$

Minimizing Eq. (14) with respect to the control force $\mathbf{B}_0^{-1/2}\mathbf{F}^c$ and utilizing the result into Eq. (13), we obtain:

$$\mathbf{B}_0^{-1/2}\mathbf{F}^c = \mathbf{B}_0^{-1/2}\mathbf{C}_s\hat{\mathbf{F}} = \mathbf{0} \tag{15}$$

Because the matrix $\mathbf{B}_0$ is positive-definite matrix, the solution of Eq. (15) satisfies:

$$\mathbf{C}_s\hat{\mathbf{F}} = \mathbf{0} \tag{16}$$

The non-trivial solution of Eq. (16) is to satisfy $\det(\mathbf{C}_s) = 0$. However, the coefficient matrix $\mathbf{C}_s$ is rank-deficiency and the solution is characterized by its singular value decomposition. It will be observed that the damage exists at the element whose internal force in any element caused by the stiffness variation is not changed. It comes from the result that the change of the stiffness by the damaged element only represents the rigid body motion. The problem is to determine the coefficient matrix $\mathbf{C}_s$ to describe the force variation and it is established by evaluating the measured displacements.

Let us assume that the constraints to describe the damaged responses are expressed such as Eq. (8). Assuming that of all admissible dynamic strain energy distributions that yield the correct displacement at the loaded coordinates, the actual strain field that satisfies force equilibrium is to minimize the variation in the dynamic strain energy, the constraint force vector can be directly derived. It can be also interpreted that the actual displacement vector of all the admissible displacement vectors that give the measured displacements by the action of the external forces as well as the additional forces is to minimize the variation in dynamic strain energy of Eq. (14).

Modifying the constraint equation of Eq. (8) by the relation of $\mathbf{B}_0\mathbf{U} = \hat{\mathbf{F}} + \mathbf{F}^c$, it follows:

$$\mathbf{A}\mathbf{B}_0^{-1/2}\mathbf{B}_0^{-1/2}\mathbf{B}_0\mathbf{q} = \mathbf{A}\mathbf{B}_0^{-1/2}\mathbf{B}_0^{-1/2}\left(\mathbf{F}^c + \hat{\mathbf{F}}\right) = \mathbf{0} \tag{17}$$

Solving Eq. (17) with respect to $\mathbf{B}_0^{-1/2}\mathbf{F}^c$ with the solution of the generalized Moore-Penrose inverse, it is derived as:

$$\mathbf{B}_0^{-1/2}\mathbf{F}^c = -\mathbf{B}_0^{-1/2}\hat{\mathbf{F}} + \left[\mathbf{I} - \left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)\right]\mathbf{y} \tag{18}$$

where '+' denotes the Moore-Penrose inverse and $\mathbf{y}$ is an arbitrary vector.

Introducing Eq. (18) into Eq. (17) and providing the minimization condition into the result, we obtain:

$$-\mathbf{B}_0^{-1/2}\hat{\mathbf{F}} + \left[\mathbf{I} - \left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)\right]\mathbf{y} = \mathbf{0} \tag{19}$$

Again, solving Eq. (19) with respect to the arbitrary vector $\mathbf{y}$, it follows:

$$\mathbf{y} = \left[\mathbf{I} - \left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)\right]\left(\mathbf{B}_0^{-1/2}\hat{\mathbf{F}}\right) + \left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)\mathbf{z} \tag{20}$$

where $\mathbf{z}$ is an arbitrary vector and $\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+ = \left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+$. Inserting Eq. (20) into Eq. (19) and arranging the result, the constraint force vector is derived as:

$$\mathbf{F}^c = -\mathbf{B}_0^{1/2}\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^+\mathbf{A}\mathbf{B}_0^{-1}\hat{\mathbf{F}} \tag{21}$$

The constraint force vector of Eq. (21) is expressed as a function of the initial stiffness matrix as well as the external force vector. It is observed that Eq. (21) satisfies the condition of Eq. (16) under any external excitation $\hat{\mathbf{F}}$. Thus, the coefficient matrix $\mathbf{C}_s$ can be defined as:

$$\mathbf{C}_s = -\mathbf{B}_0^{1/2}\left(\mathbf{A}\mathbf{B}_0^{-1/2}\right)^{+}\mathbf{A}\mathbf{B}_0^{-1} \tag{22}$$

Because $\mathbf{C}_s$ is rank-deficiency, the damage locates at the element whose internal force is the same from the singular vectors by SVD[1] on the coefficient matrix of Eq. (22). This method does not need any information on the input force data and must be a simple method, but it requires the exactly measured data. In the following, we consider the displacement variation between the intact and damaged systems instead of the force variation.

The variation in the strain energy of Eq. (14) can be modified as:

$$\delta U = \frac{1}{2}(\Delta\mathbf{U})^T \mathbf{B}_0(\Delta\mathbf{U}) \tag{23}$$

Assuming that the external forces are invariant, the equilibrium equations of undamaged and damaged systems are related as:

$$\mathbf{B}_0\hat{\mathbf{U}} = \mathbf{B}\mathbf{U} \tag{24}$$

Inserting the relations of $\mathbf{B} = \mathbf{B}_0 + \Delta\mathbf{B}$ and $\mathbf{U} = \hat{\mathbf{U}} + \Delta\mathbf{U}$ into Eq. (24), neglecting the higher order term and arranging the result, it follows that:

$$\mathbf{B}_0^{1/2}(\Delta\mathbf{U}) = -(\Delta\mathbf{B})\hat{\mathbf{U}} \tag{25}$$

Considering the condition to minimize Eq. (23), the left-hand side of Eq. (25) is written as:

$$\mathbf{B}_0^{1/2}(\Delta\mathbf{U}) = \mathbf{B}_0^{1/2}\mathbf{C}_f\hat{\mathbf{U}} = \mathbf{0} \tag{26}$$

where $\mathbf{C}_f = -\mathbf{B}_0^{-1/2}(\Delta\mathbf{B})$ and it is the coefficient matrix of the variation vector in the displacement between the intact and damaged systems.

Equation (26) indicates that the damage exists at the element whose internal deformation is zero such that the element represents the rigid body motion. Utilizing the measured displacements of Eq. (8) and taking the same process to obtain the relation like Eq. (22), the displacement variation can be derived. The constraints by measured data of Eq. (8) can be modified as:

$$\mathbf{A}\left(\hat{\mathbf{U}} + \Delta\mathbf{U}\right) = \mathbf{0} \quad \text{or} \quad \mathbf{A}(\Delta\mathbf{U}) = -\mathbf{A}\hat{\mathbf{U}} \tag{27}$$

and Eq. (27) is also modified as:

$$\mathbf{A}\mathbf{B}_0^{-1/2}\left[\mathbf{B}_0^{1/2}(\Delta\mathbf{U})\right] = -\mathbf{A}\hat{\mathbf{U}} \tag{28}$$

Solving Eq. (28) with respect to $\mathbf{B}_0^{1/2}\left(\Delta\hat{\mathbf{U}}\right)$, it follows that:

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[1] Singular value decomposition takes a rectangular matrix of gene expression data. The SVD theorem states:

$$\mathbf{A}_{n\times p} = \mathbf{U}_{n\times n}\mathbf{S}_{n\times p}\mathbf{V}^T{}_{p\times p}$$

where the columns of **U** are the left singular vectors (gene coefficient vectors); **S** (the same dimensions as **A**) has singular values and is diagonal (mode amplitudes); and $\mathbf{V}^T$ has rows that are the right singular vectors (expression level vectors). The SVD represents an expression of the original data in a coordinate system where the covariance matrix is diagonal.

,

$$\mathbf{B}\quad(\Delta\mathbf{U}) = -(\mathbf{AB}^{-}\quad)^{+}(\mathbf{AU}) + \left[\mathbf{I} - (\mathbf{AB}^{-}\quad)^{+}(\mathbf{AB}^{-}\quad)\right]\mathbf{s} \qquad (\quad)$$

$\mathbf{s}$ ( ) ( )

$$\mathbf{s} = -\left[\mathbf{I} - \mathbf{AB}^{-}\quad{}^{+}\mathbf{AB}^{-}\quad\right]\left[\mathbf{AB}^{-}\quad{}^{+}\mathbf{AB}^{-}\quad\right] + \left[\mathbf{AB}^{-}\quad{}^{+}\mathbf{AB}^{-}\quad\right]\mathbf{r}$$
$$= \left[\mathbf{AB}^{-}\quad{}^{+}\mathbf{AB}^{-}\quad\right]\mathbf{r} \qquad (\quad)$$

$\mathbf{r}$ ( ) ( )

$$\Delta\mathbf{U} = -\mathbf{B}^{-}\quad(\mathbf{AB}^{-}\quad)^{+}\mathbf{AU} \qquad (\quad)$$

( ) ( ) $\mathbf{C}_f$

$$\mathbf{C}_f = -\mathbf{B}^{-}\quad(\mathbf{AB}^{-}\quad)^{+}\mathbf{A} \qquad (\quad)$$

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3. A dynamic system with five degrees of freedom

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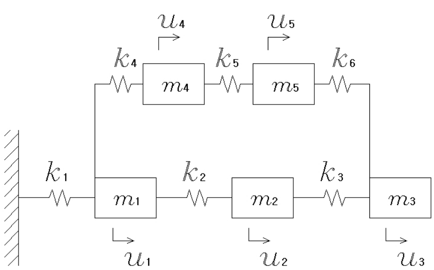

Fig. 2.

$$\omega_1^2 = 11.748 \text{ rad/sec}, \quad \mathbf{\Phi}_1 = \begin{bmatrix} 0.1644 & 0.2338 & 0.2601 & 0.2263 & 0.2635 \end{bmatrix}^T \tag{33}$$

Inserting the above measured data and stiffness matrix into Eq. (32) and taking the SVD on the results, we obtain the numerical results listed in Tables 1 and 2, which exhibit that the damages are detected by investigating the variation vector of modal displacement and force in the singular matrix **U** or $\mathbf{V}^T$ corresponding to zero element in singular value matrix **S**. It is displayed that the damage is located at the element between two nodes that the absolute values of internal force or deformation are not changed.

Table 1. Numerical results of SVD of $\mathbf{C}_f$ (k_2 and k_5 damage)

U	-0.4097	0.1173	-0.0399	-0.8466	-0.3163
	0.3999	-0.7533	0.2244	-0.1404	-0.4500
	-0.0064	0.4622	0.6976	0.2213	-0.5006
	0.5737	0.4028	-0.5638	-0.0325	-0.4355
	-0.5857	-0.2069	-0.3787	0.4621	-0.5071
S	78.2839	0	0	0	0
	0	1.0000	0	0	0
	0	0	1.0000	0	0
	0	0	0	1.0000	0
	0	0	0	0	0.0000
$\mathbf{V}^T$	-0.3216	0.1173	-0.0399	-0.8466	0.4057
	-0.4449	-0.7533	0.2244	-0.1404	-0.4057
	-0.5006	0.4622	0.6976	0.2213	-0.0000
	-0.4281	0.4028	-0.5638	-0.0325	-0.5792
	-0.5145	-0.2069	-0.3787	0.4621	0.5792

Table 2. Numerical results of SVD of $\mathbf{C}_s$ (k_2 and k_5 damage)

U	-0.3216	0.0000	0.0000	0.8556	0.4057
	-0.4449	0.0181	0.7979	0.0251	-0.4057
	-0.5006	-0.8056	-0.2550	-0.1882	0.0000
	-0.4281	0.4124	-0.5461	0.1137	-0.5792
	-0.5145	0.4251	0.0127	-0.4680	0.5792
S	78.2839	0	0	0	0
	0	1.0000	0	0	0
	0	0	1.0000	0	0
	0	0	0	1.0000	0
	0	0	0	0	0.0000
$\mathbf{V}^T$	-0.4097	0	-0.0000	0.8556	-0.3163
	0.3999	0.0181	0.7979	0.0251	-0.4500
	-0.0064	-0.8056	-0.2550	-0.1882	-0.5006
	0.5737	0.4124	-0.5461	0.1137	-0.4355
	-0.5857	0.4251	0.0127	-0.4680	-0.5071

4. Conclusions

The structural damage can be detected from the information on the variation in displacement or force vector before and after the occurrence of damage. Minimizing the variation in the dynamic strain energy with respect to the force or displacement variation and taking the SVD on the coefficient matrices of the force or displacement variation, we straightforwardly provided the damage detection method. It was observed that the damages located at the elements whose internal forces or deformations between two adjacent nodes of finite element model are the same in the singular matrix. The methods can be widely utilized in detecting the single or multiple damages. But it is necessary to modify or refine it for applying more generally to the systems with less measurement data, instrumental noise or measurement errors.

Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) Grant funded by the Korea Government (MEST) (No. 2011-0012164).

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