726. The analysis of a composite beam with piezoelectric actuator based on the approximate method

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Abstract. The paper presents application of an approximate Galerkin method in analysis of a mechatronic system with piezoelectric actuator. The considered mechatronic system is a composed beam made of steel and piezoelectric transducer. The transducer is used as an actuator supplied by an external voltage source. A characteristic that describes relation between displacement of the system's free end and amplitude and frequency of the applied voltage was assigned using the approximate method. In the considered system a Macro Fiber Composite – MFC is used as the actuator in order to generate vibration. This work is connected with the previous publications concerned with analysis of mechatronic systems with piezoelectric transducers used as actuators or as vibration dampers with external shunting network [1-7].

Keywords: approximate method, piezoelectric actuator, smart material, macro fiber composite.

Introduction

Applications of piezoelectric sensors and actuators are getting more and more popular in many kinds of technical devices. Piezoelectric transducers are widely used as passive or active vibration dampers, sensors for example in structural health monitoring applications or as actuators [5, 9, 11]. Such a large number of applications of piezoelectric materials is possible because both the direct or reverse piezoelectric effects are used. In this paper application of reverse piezoelectric effect in mechatronic systems is presented – the piezoelectric transducer is used as an actuator supplied by the external voltage source in order to generate system's vibration. The development of piezoelectric materials is observed too. New piezoelectric transducers are being developed such as Macro Fiber Composites (MFC). Main benefits of the MFC given by the manufacturer are: increased strain actuator efficiency, damage tolerance, environmentally sealed packages, available as elongators and contractors. The Macro Fiber Composite was invented by NASA in 1996. It consists of rectangular piezo ceramic rods sandwiched between layers of adhesive, electrodes and polyimide film [12, 13]. The development of transducers allows new applications of piezoelectric materials and improved operation of existing devices.

To design a system with piezoelectric transducer used as actuator or sensor it is important to use precise mathematical model of it in order to obtain required dynamic characteristic of this system. This is why in previous works a series of mathematical models of systems with piezoelectric actuators or passive vibration dampers were described and computer methods were used to analyse systems with piezoelectric materials [1, 2, 8]. The approximate Galerkin method was used in order to calculate dynamic characteristics of considered mechatronic systems. It was verified for its accuracy and corrected [3, 4]. It was proved that the approximate Galerkin method can by used to analyze such kind of systems and after correction obtained results can be treated as very precise. It was also proved that inexactness of the approximate Galerkin method depend only on the system's boundary conditions [6].

In this work the corrected approximate Galerkin method is used to analyse system with MFC actuator.

Description of the considered system

The considered system is presented in Fig. 1. It is a cantilever bending beam with a MFC actuator bonded on the overall length of the beam's upper surface. There is an assumption that the actuator is perfectly bonded to the beam's surface, so the beam and actuator's strains are exactly the same. Influence of the glue layer between the beam and transducer was neglected. The MFC actuator is supplied by an external harmonic voltage source described by equation:

$$U(t) = U_0 \cos \omega t. \tag{1}$$

The main aim of this work is to designate a dynamic characteristic α_V that describes relation between amplitude of the beam's free end vibration and parameters of the electric voltage that supplied the piezoelectric actuator [5, 7]. This characteristic is described by equation:

$$y(x,t) = \alpha_V \cdot U(t). \tag{2}$$

Symbol $y(x, t)$ denotes the beam's deflection.

In order to analyze vibration of the system following assumptions were made:

- material of which the system is made is subjected to Hooke's law,
- the system has a continuous, linear mass distribution,
- the system's vibration is harmonic,
- planes of cross-sections that are perpendicular to the axis of the beam remain flat during deformation of the beam – an analysis is based on the Bernoulli's hypothesis of flat cross-sections,
- displacements are small compared with the dimensions of the system.

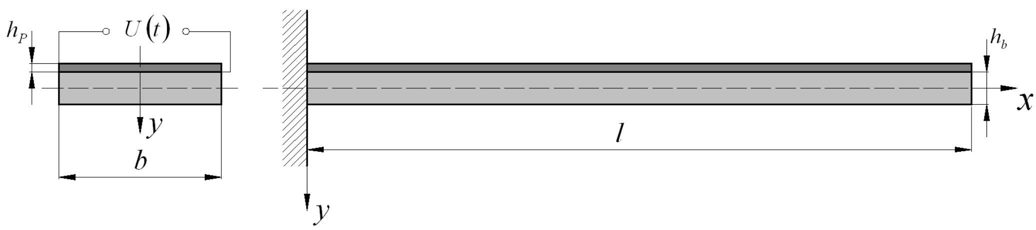

Fig. 1. The considered system with MFC actuator

A mathematical model and the system's analysis

The analysis of the considered system was based on the discrete – continuous mathematical model where the beam was described taking into account continuous distribution of the beam's mass and MFC actuator supplied by the external voltage source was tread as RC electric circuit and described using corresponding electric circuit equation [10]. An equation of the beam's motion was assigned in agreement with d'Alembert's principle taking into account arrangement of forces and bending moments acting in the system that are presented in Fig. 2. Symbols $T(x, t)$ and $M(x, t)$ denote transverse forces and bending moments that replace action of the cut-off part of the beam. $M(x, t)$ is a bending moment generated by the actuator as a result of the externally applied voltage. In agreement with constitutive equations of piezoelectric materials this bending moment can be described by equation:

$$M_p(x,t) = \frac{1}{2}(h_b + h_p)\cdot c_{11}{}^E A_p \left[S_1(x,t) - \lambda_1(t)\right], \tag{3}$$

where: $c_{11}{}^E$ and A_p are Young's modulus at constant field and cross-section area of the MFC actuator. S_1 is the beam's surface strain and λ_1 is the free actuator's strain occurs as a result of externally applied voltage:

$$\lambda_1(t) = \frac{d_{33}}{h_p}\cdot U(t), \tag{4}$$

where d_{33} is the actuator's piezoelectric constant. Obtained equation of the beam's motion is:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -a^4\left(1+\eta_b \frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{(h_b + h_p)\cdot c_{11}{}^E A_p}{2\rho_b A_b}\cdot\frac{\partial^2}{\partial x^2}\left[S_1(x,t) - \lambda_1(t)\right], \tag{5}$$

where:

$$a = \sqrt[4]{\frac{E_b J_b}{\rho_b A_b}}. \tag{6}$$

Symbols E_b, J_b, ρ_b, A_b are the beam's Young's modulus, moment of inertia, density and cross-section area. Taking into account that the thickness of the MFC actuator is the much smaller than the thickness of the beam its impact on the beam's stiffness and mass were neglected in the equation (5). Rheological properties of the beam were introduced using Kelvin – Voigt model of the beam's material. The beam's Young's modulus was replaced by substitute modulus E_b^* described by equation:

$$E_b{}^* = E_b\left(1+\eta_b\frac{\partial}{\partial t}\right), \tag{7}$$

where η_b is the retardation time [11].

Internal resistance R_P and capacitance C_P of the piezoelectric actuator were taken into consideration so it was described by equation:

$$R_P C_P \frac{\partial U_C(t)}{\partial t} + U_C(t) = U(t), \tag{8}$$

where:

$$U_C(t) = \frac{|U_0|}{\omega C_P |Z|}\cdot\sin(\omega t + \varphi), \tag{9}$$

is electric voltage on the capacitor C_P. $|Z|$ and φ are the modulus and argument of an electrical impedance. Analysis of the considered system was based on the corrected approximate Galerkin method. Equation of the beam's deflection was assumed as:

$$y(x,t) = A \cdot \sin(k_n x)\cos(\omega t), \tag{10}$$

where:

$$k_n = \frac{(2n-1)\pi}{2l}, \qquad n = 1,2,3... \tag{11}$$

A denotes amplitude of the beam's vibration. Assumptions and verification of the approximate Galerkin method were presented in details in the previous works [3, 4].

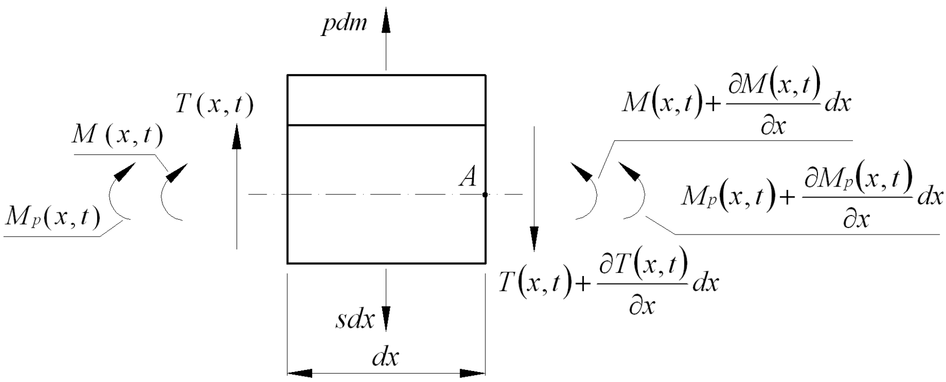

Fig. 2. Arrangement of forces and bending moments acting in the system

Obtained results

Using discrete – continuous mathematical model of the considered system and corrected approximate Galerkin method the modulus of dynamic characteristic, marked Y_V was calculated. Geometrical and material parameters of the system are presented in Table 1.

Table 1. The considered system's properties

$l = 0{,}112[m]$	$h_p = 0{,}0003[m]$
$b = 0{,}04[m]$	$d_{33} = 460[pmV^{-1}]$
$h_b = 0{,}002[m]$	$c_{11}{}^E = 30{,}336[GPa]$
$h_p = 0{,}0003[m]$	$\rho_p = 5440[kg \cdot m^{-3}]$
$\rho_b = 7850[kg \cdot m^{-3}]$	$C_p = 5{,}7[nF]$
$E_b = 210[GPa]$	$R_p = 50[\Omega]$
$\eta_b = 8 \cdot 10^{-5}[s]$	

The cantilever beam is made of steel and MFC actuator that was selected from the catalogue of the Smart Material Company [13]. It is M-8528-P1 actuator with expanding motion.

Taking into account geometrical and material parameters of the considered system the modulus of dynamic characteristic Y_V was calculated and presented in Fig. 3. Obtained results are presented in a half logarithmic scale for the first three system's natural frequencies. The considered mechatronic system's resonance zones are consistent with resonance zones of the mechanical subsystem (the beam without piezoelectric actuator).

Conclusions

Using the corrected approximate Galerkin method it is possible to analyse such kind of mechatronic systems with Macro Fiber Composite actuators. Obtained dynamic characteristic describes relation between electric voltage supplied actuator and amplitude of the system's vibration. Using MFC actuators it is possible to obtain better system's operation than using PZT transducers (analysis of systems with PZT actuators was presented in other paper [5, 7]). What is more Macro Fiber Composites are much thinner and lighter and have better piezoelectric properties (higher value of the piezoelectric constant). MFC actuators are also flexible and durable and are protected against the influence of the environmental hazards. In future works described method of the system analysis will be used to designate influence of all system's parameters on its dynamic characteristic. Mathematical model will be developed and influence of the glue layer between MFC actuator and the beam's surface will be taken into consideration.

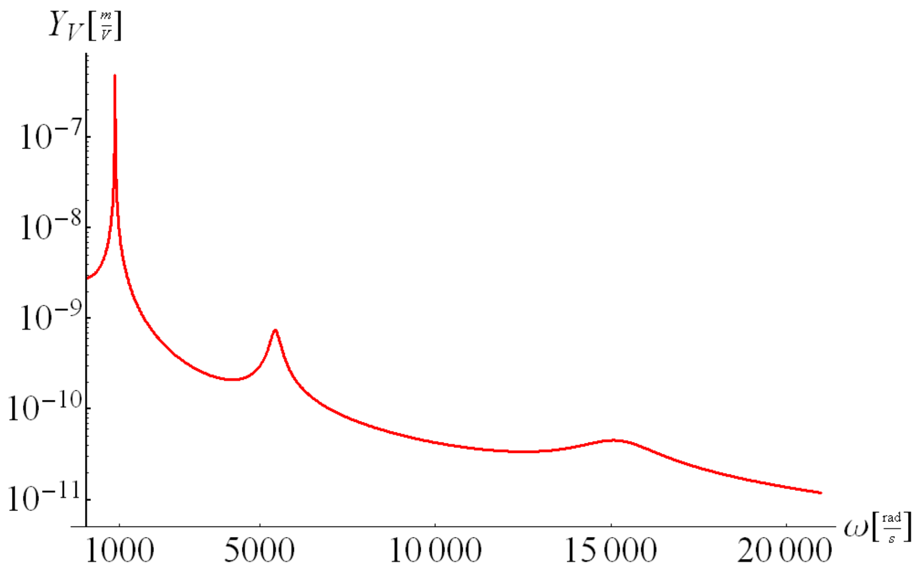

Fig. 3. Absolute value of the dynamic characteristic of system with piezoelectric actuator, for the first three natural frequencies

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