672. Some problems of nonlinear precise vibromechanics and vibroengineering (summary)

Kazimieras Ragulskis
Kaunas University of Technology, Lithuania
E-mail: kazimieras3@yahoo.com
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Abstract. The investigations of the author in the field of precise vibromechanics and vibroengineering are overviewed. Some effects and qualities of nonlinear systems as well as developments of new principles of operation thereof are considered. Various designs of mechanical systems of different types, new principles, innovations and more than 1750 inventions and patents have been developed with co-authors.

Keywords: vibrations, waves, non-linear effects, new principles, vibromotors, vibrorobots, vibrocontrol.

Introduction

Mechanical systems undergo fast development because of their interaction with various physical processes, magnetism, electrical and variable gravitational fields, chemical and life processes and also because of changes according to their size, dynamicity, velocities, precision directly into unachievable limits.

At the small scale one has already investigate objects of nanometer size. Soon will be the time when humanity will have to work with bodies which are smaller thousands and even more times than the nanometer. On the other side, especially for the research and use of objects beyond the limits of the earth systems of the dimensions of hundreds of meters are to be created and in the future their size is to reach up to thousands of kilometers.

Systems expand also because of their motion at big velocities acting not only in the conditions of our earth, but also in space. High processes of dynamicity of some systems are required. On the other side, in some applications in crystallography, in the investigations of live organisms systems moving very slowly are required.

Despite of the sizes and velocities there are requirements to substantially increase the stability of the systems, compatibility with discrete microelectronics and controllability, precision in positioning of bodies in space, performing trajectories and laws of motions, ensuring desirable structures and geometrical parameters.

From the point of view of precise mechanics in engineering everything, in the non-living nature as well as in the living one, performs vibrational and wave motions. These are complicated problems in the development of investigations in order to reveal the effects and qualities of non-linear dynamical systems, to create scientific basis and principles for the construction of enhanced new systems. A short overview on some contributions to the field of vibroengineering is presented in this paper.

1. Self organization in some mechanical systems

Essentially non-linear systems. In many cases mechanical systems operate far from resonances. For the investigation of dynamics of such systems an approximate analytical method was created, which is based on the division of motion into slow and fast. This enables to determine the conditions of existence and stability of the stationary regimes. The method is suitable for the investigation of quasi-linear cases. It was successfully used for the investigation of simple synchronization of dynamical systems.
The modified asymptotic method had to be developed not only for the cases of quasi-linear, but also essentially non-linear dynamical systems. Parameters of stationary regimes, conditions of their existence and stability, regions of validity and attraction have been obtained. This method justified itself in investigation of problems of systems of different types.

In non-linear systems stationary motions of various types may exist. But here only local motions which are important in practical applications are analyzed, namely the motions of the systems of rotor or stepwise types, in which the directed motion about the constant one is accompanied by vibrations. Here systems with one degree of freedom are analyzed, but they are easily generalized for systems having a higher number of degrees of freedom and also for autonomous systems.

As an example further a rotating system with one degree of freedom is described with the differential equation of motion of the following type:

\[ \ddot{\phi} = \Phi(\phi, \dot{\phi}, \omega t) + M(\dot{\phi}), \]  
(1)

where \( \phi \) is the angle of rotation, \( \Phi \) is a periodic function of \( \phi \) and \( t, \dot{t} = \frac{d}{dt} \), \( M \) is the moment of external and dissipative forces, which is a periodic function of \( \phi \).

For the investigation in the equation the following substitution is performed:

\[ \phi = \Omega t + \bar{\phi} + \tilde{\phi}, \]  
(2)

where \( \Omega \) is the frequency of steady state motion, \( \bar{\phi} \) is the slow motion, \( \tilde{\phi} \) is the fast motion.

The equation (1) with account of equation (2) is rearranged into the following one:

\[ \ddot{\bar{\phi}} + \dot{\bar{\phi}} = \Phi\left(\Omega t + \bar{\phi} + \tilde{\phi}, \Omega + \bar{\phi} + \dot{\bar{\phi}} + \ddot{\phi}, \omega t\right) + \epsilon^\alpha M\left(\Omega + \bar{\phi} + \dot{\bar{\phi}}\right), \]  
(3)

where \( \epsilon \) is a dummy small parameter which depends on the sought dynamically synchronized motion is chosen in the process of calculations, \( \alpha \geq 0 \).

Cases of combined dynamical synchronization are analyzed when:

\[ n\Omega = m\omega, \]  
(4)

where \( m, n \) are integer numbers and their ratio:

\[ \left| \frac{m}{n} \right| \leq 1. \]  
(5)

For the investigation of fast motion linear equation with respect to \( \tilde{\phi} \) and its derivatives is assumed, while \( \bar{\phi} \) is considered a constant quantity:

\[ \ddot{\phi} = \Phi - \bar{\Phi} = \Phi\left(\Omega t + \bar{\phi} + \tilde{\phi}, \Omega + \bar{\phi} + \dot{\bar{\phi}} + \ddot{\phi}, \omega t\right) - \bar{\Phi} = \ddot{\Phi}, \]  
(6)

where a horizontal dash over the letters means averaging with respect to time, wave over the letters means a variable quantity with an average with respect to time equal to zero.

From the equation (6) \( \dot{\phi} \) is found.

In order to obtain an equation for the determination of slow motion the averaging of the right side of equation (3) with respect to time \( t \) is performed. This is performed by expanding \( \Phi \) into a power series with respect to \( \bar{\phi}, \tilde{\phi}, \ddot{\phi} \) and taking into account several first members of the power series and also with a last member and the averaged moment of the dissipative and external forces \( M \), the differential equation for the determination of the slow process \( \bar{\phi} \) is obtained:

\[ \bar{\phi} = F(\Omega, \bar{\phi}, \tilde{\phi}, \ddot{\phi}). \]  
(7)
By assuming $\bar{\phi} = \tilde{\phi} = 0$ in the nonlinear part of the equation, the following equation is obtained:

$$F(\Omega, n\Omega - m\omega, \bar{\phi}) = 0,$$

which is used for the determination of the values of $\bar{\phi}$ in the process of the stationary motion.

The existence of real solutions of $\bar{\phi}$ is the condition of existence of the analyzed regime of combined synchronization $n\Omega - m\omega = 0$.

Which of the existing regimes are stable is determined by the conditions of stability:

$$\frac{\partial F}{\partial \bar{\phi}} < 0. \quad (9)$$

Regimes of this type are valid when:

$$\left| \frac{\partial F}{\partial \bar{\phi}} \right| << \omega. \quad (10)$$

Equation (7) in the small vicinity of the stationary regime approximately determines the region of attraction. The same results are also obtained by the method of small parameter.

It is very complicated to create mathematical models of separate parts of some systems (coauthor). For the solution of such problems the methods based on simultaneous use of experimental methods and computer calculations have been proposed. The methods justified themselves in performing investigations in the systems of type “living organism – technology” or in systems in which some parameters of some parts in dynamical regimes undergo large variations.

In many systems qualitative criterions, such as the existence, stability of the process and others, are determined not by all the processes and parameters, but only by the leading ones. This enables to achieve reliable and visual results after essential simplification of the models of the analyzed system and using simplified analytical or analytical – numerical methods.

The science of vibrations and waves of non-linear systems is of general character, not of functional purpose. Due to this it is applicable in various fields. In particular, investigations related with definite systems are the efforts of not one researcher, but of their group. Thus, the investigations had to be performed not only with specialists in mechanics, but also of other fields (mathematicians, physicists, electrical and control engineers, biologists, etc.).

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Synchronization. Simple dynamical synchronization is mainly the phenomenon of quasi-linear systems, while combined synchronizations are the phenomena of essentially nonlinear systems. Thus for the investigation of the latter systems the method of division of motions into slow and fast and the method of small parameter were proposed. Rotational and translational motions about which vibrations take place were analyzed. Here the results for the most simplest cases are presented for better visibility and economy of space.

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Dynamics of essentially nonlinear systems. Combined dynamical synchronization. The differential equation of motion of a mechanical system with one degree of freedom on a vibrating basis is the following one:

$$I(\phi)\ddot{\phi} + 0.5I'(\phi)\dot{\phi}^2 + x^2N_x(\phi) + y^2N_y(\phi) + \tilde{\psi}N_{\psi1}(\phi) + \tilde{\psi}^2N_{\psi2}(\phi) +$$

$$+ \chi\psi N_{x\psi}(\phi) + \chi\psi N_{y\psi}(\phi) + gN_{\rho}(\phi) = M_{\phi}(\phi, \dot{\phi}), \quad (11)$$

where $\phi$ is the turning angle,

$x, y, \psi$ are orthogonal and rotational vibrations of the foundation as given periodic functions of $\omega t$,

$' = d/dt, \ ' = d/d\phi,$
\( I(\varphi) \) is the moment of inertia,
\( M(\dot{\varphi}) \) is the moment of external and dissipative forces,
all other functions are periodic with respect to \( \varphi \).

The following system is analyzed:
\[
\ddot{\varphi} + (a_1 + a_2 \sin \omega t) \sin \varphi + h\dot{\varphi} = 0. \tag{12}
\]
After performing the substitution:
\[
\varphi = \Omega t + \bar{\varphi} + \check{\varphi}, \tag{13}
\]
where \( \Omega \) is the velocity of stationary motion,
\( \bar{\varphi} \) and \( \check{\varphi} \) are the slow and fast motions respectively,
the stationary motions of combined synchronization are sought:
\[
m\omega = n\Omega, \tag{14}
\]
where \( m = \pm 1, \pm 2, \ldots ; n = \pm 1, \pm 2, \ldots , \)
\[
\left| \frac{m}{n} \right| \leq 1. \tag{15}
\]

It is clear that when \( m=n=1 \) simple dynamic synchronization takes place and when \( m\neq n \neq 1 \) combined synchronization takes place. Investigations of the latter type of synchronization are related with the essentially nonlinear differential equation.

First the method of division of motion into slow and fast is presented.
Separate differential equations are obtained:
a linear one for the fast motion:
\[
\ddot{\bar{\varphi}} + (a_1 + a_2 \sin \omega t) \sin (\Omega t + \bar{\varphi}) - a_2 \sin \omega t \sin (\Omega t + \varphi) = 0, \tag{16}
\]
and a nonlinear autonomic one for the slow motion:
\[
\ddot{\check{\varphi}} + (a_1 + a_2 \sin \omega t) \sin (\Omega t + \bar{\varphi} + \check{\varphi}) + h\Omega = 0. \tag{17}
\]
From the equation (14) it is obtained:
\[
\check{\varphi} = \frac{a_1}{\Omega^2} \sin (\Omega t + \bar{\varphi}) + \frac{a_2}{2(\omega - \Omega)^2} \cos \left[ (\omega - \Omega) t - \varphi \right] - \frac{a_2}{2(\omega + \Omega)^2} \cos \left[ (\omega + \Omega) t + \bar{\varphi} \right]. \tag{18}
\]
In order to perform the averaging of the equation the nonlinear term is expanded into a power series with respect to \( \beta \) and then averaging is performed, that is:
\[
\overline{\ddot{\varphi}} + (a_1 + a_2 \sin \omega t) \left[ \sin (\Omega t + \bar{\varphi}) + \bar{\varphi} \cos (\Omega t + \bar{\varphi}) - \frac{1}{2!} \bar{\varphi}^2 \sin (\Omega t + \bar{\varphi}) - \ldots \right] + h\Omega = 0. \tag{19}
\]
The problem is how many members of the power series with respect to \( \bar{\varphi} \) are to be taken into account.

The case of combined synchronization:
\[
\omega = 2\Omega. \tag{20}
\]
In this case in the equation of slow motion (19) the first two members of the power series with respect to \( \bar{\varphi} \) are to be taken into account, that is:
\[
\ddot{\bar{\varphi}} + (a_1 + a_2 \sin \omega t) \left[ \sin (\Omega t + \bar{\varphi}) + \bar{\varphi} \cos (\Omega t + \bar{\varphi}) \right] + h\Omega = 0. \tag{21}
\]
After performing the averaging with respect to \( t \) the equation (20) takes the following form:
\[
\ddot{\bar{\varphi}} + \frac{a_1 a_2}{8(\omega - \Omega)^2} \cos 2\bar{\varphi} + h\Omega = 0. \tag{22}
\]
Further it is denoted:
\[
\bar{\varphi} = \bar{\varphi}_+ + \bar{\varphi}_-, \tag{23}
\]
where \( \bar{\phi}_- = \text{const} \) is the value of stationary motion, \( \bar{\phi}_- \) is varying slowly with respect to \( \bar{\phi} \).

From the equation (22) by taking into account the equations (20), (23) the equation for the determination of \( \bar{\phi}_- \) is obtained:

\[
\frac{a_1 a_2}{\omega^2} \cos 2\bar{\phi}_- + h\omega = 0. \tag{24}
\]

In the process of stationary motion:

\[
\bar{\phi} = \bar{\phi}_-, \quad \bar{\phi}_- = 0. \tag{25}
\]

The conditions of existence and stability of the motion of the analyzed type respectively are the following ones:

\[
\left| \frac{h\omega^3}{a_1 a_2} \right| < 1, \tag{26}
\]

\[
-2\frac{a_1 a_2}{\omega^2} \sin 2\bar{\phi}_- > 0. \tag{27}
\]

Motions of the analyzed type exist when:

\[
\omega^2 >> -2\frac{a_1 a_2}{\omega^2} \sin 2\bar{\phi}_-. \tag{28}
\]

The slow motions in the vicinity of the stationary regime according to the equations (22), (23) are determined by the following equation:

\[
\bar{\phi}_- + 0.5\frac{a_1 a_2}{\omega^2} \cos 2(\bar{\phi}_- + \bar{\phi}_-) + h\bar{\phi}_- + h\frac{\omega}{2} = 0, \tag{29}
\]

or after performing the expansion of \( \cos 2(\bar{\phi}_- + \bar{\phi}_-) \) into a power series with respect to \( \bar{\phi}_- \) it is obtained:

\[
\bar{\phi}_- - \frac{a_1 a_2}{\omega^2} \left( \bar{\phi}_- \sin 2\bar{\phi}_- + \frac{1}{2!} \bar{\phi}_-^2 \cos 2\bar{\phi}_- - \cdots \right) + h\bar{\phi}_- = 0. \tag{30}
\]

According to the linear part of the latter equation the transient process in the vicinity of the stationary point is easily determined.

For the investigation of essentially nonlinear rotational systems the small parameter is also used for the case of small dissipative forces.

In the equation (12) it is assumed:

\[
\bar{\phi} + \varepsilon (a_1 + a_2 \sin \omega t) \sin \phi + \varepsilon^k h\bar{\phi} = 0, \tag{31}
\]

where \( \varepsilon \) is a small parameter, \( k \geq 1 \) is a constant quantity.

In steady state motion the same results are obtained which were obtained by using the method of division of motion into slow and fast. For example in the case determined by equation (20) it is necessary to assume:

\[
k = 2. \tag{32}
\]

When the dissipative forces are large it is impossible for the motions of combined synchronization to take place in the system.

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**Experimental determination of the effect of synchronization.** We proposed a simple method for the analysis of synchronous motions. This is simpler than the method of film taking. Here the analysis of translational and rotational motion is shown for the case when an unbalanced rotor is forced to rotate because of the harmonic vibrations of its foundation in the direction of one axis (Fig. 1). A black screen is attached to the rotor perpendicularly to its axis.
of rotation which is indicated instead of the black color by dashes in two directions. On the
black screen two dots which reflect light with high quality are painted: one of them in the center
O which coincides with the axis of rotation of the rotor and another one A in the periphery. In
the process of steady state motion the point O moves according to the line, while the point A
describes a closed trajectory with \(2(n+1)\) extreme points, from which \((n+1)\) are maximum points
and \((n+1)\) are minimum points. The images photographed with large exposition are shown in
Fig. 2, Fig. 3, Fig. 4 at \(n = 1\), \(n = 2\) and \(n = 3\) respectively. When the velocities are sufficient
because of the inertia of the eyes the processes of synchronization are well visible without
photographing.

![Fig. 1. The screen with the white points O and A for the observation of complicated motions](image1)

![Fig. 2. Topological image of the stationary motion \(n = 1\) of the rotor with the unbalanced mass arising from the vibration of the foundation](image2)

![Fig. 3. Topological image of the stationary motion \(n = 2\) of the rotor with the unbalanced mass arising from the vibration of the foundation](image3)

![Fig. 4. Topological image of the stationary motion \(n = 3\) of the rotor with the unbalanced mass arising from the vibration of the foundation](image4)

The method is successfully applicable for the observation of simple and combined
synchronization of complicated systems and for the determination of limits of existence of synchronous and chaotic regimes. When the structures for which the axes are not parallel are
analyzed, then mirrors and other measures are used. This method is applicable also for control
of motions of other types.
Dynamical synchronization may exist in technical devices, vibrotechnologies, nonliving and living nature. In some cases it is useful and in some cases harmful.

New problems in the field of synchronization have been formulated, where besides of the mechanical excitations forces and links of various physical nature act.

For the synchronization of rotor systems (with coauthors) besides of the kinematical methods the dynamical ones are to be applied. Systems of new type, which enable to implement combined synchronization, transmission of motion without stages, systems with prescribed desirable phase between the rotors in the stationary regime have been proposed.

The object of the investigation is to reduce rotor vibrations and the forces causing them. One of the most effective ways for suppression of vibrations is the improvement of machine design by introduction of assembly elements resistant to vibrations. Rotary motion transmission and stabilization devices are applied for these purposes. Different types of clutches and vibration dampers suppress the vibration of constituent links of the elements of machines and mechanisms.

Thus two different methodologies may be used to avoid harmful vibrations, i.e. either development of mechanisms with low vibration activity (what may be very expensive), or installation of special constructional assemblies in the machines which are able to suppress and absorb harmful vibrations.

Elastic clutches with nonlinear characteristics are most often used in the coupled rotor systems. New clutch constructions based on the interaction between the forces of rotating masses and elastic elements are worth of special attention. In this case the radial stiffness of the shaft coupling device is decreased. Thus development of new rotary motion transmission and stabilization devices, their metrical synthesis and optimization of parameters of elastic element through optimal location and orientation of their elements enable to obtain good characteristics of the devices as well as good indices of their exploitation.

Dynamic characteristics of rotor systems depend on the parameters of their elements and rotary motion transmission and stabilization devices. Although there is a great number of various rotary motion transmission equipment designed, but most of them are not effective as far as the vibration protection in a wide range of frequencies is concerned. The connection of moving links of a rotor system is known to be performed by means of devices functioning on the effect of centrifugal forces of rotating masses. One additional degree of freedom is taken into account when solving the peculiarities of construction of such devices and comparing them with the most simple clutches. Such methodology is not reasonable in the case when the rotary motion is to be stabilized, while the mass and dimensions of the mechanical system are to be minimized. In such cases the devices in which the effect of reconstruction of centrifugal force equilibrium is applied together with the effect of reconstruction forces of vibration isolating elastic couplings are preferred.

Establishment of parameters of elastic centrifugal clutches is a complicated task involving numerous criteria and parameters. Investigation of the dynamic structure of clutches enables to choose the combination of initial parameters of the driving and driven links and to achieve endurance of better stabilization of rotary motion and of vibration isolation.

Dynamical synchronization of separate systems is based on the fact that their feeding source is limited by a common device. The purpose is to create the auto-vibrating contour of the feeding source. This enables to synchronize and achieve compatibility of the desired phase differences.

Thus there are possibilities to dynamically synchronize the systems not according to the vibrations of the foundation. The feeding source of the system 1 through the limiter of energy 2 is made into a source of limited power 3, which feeds the systems a, b, c, … . Because each of the systems a, b, c, … loads the source 3 with variable load, power pulsations in the source 3
arise. Those pulsations may force the power of the source 3 to perform pulsations at a single frequency, that may synchronize the motions of the systems a, b, c, … . Some cases have been investigated, conditions of existence and stability of synchronization and regions of validity of the solutions have been determined.

The method for dynamical synchronization of standing and travelling waves in separate bodies has been created (coauthor).

In the devices for multiple drawing of wire (coauthor) after introduction of small changes the synchronization of longitudinal vibrations of the wire has been implemented. This enabled to reduce the longitudinal vibrations of the wire several times and to improve the quality of products.

The method for the conversion of flow (of water or air) energy into electrical energy by using auto-vibrations has been proposed (coauthor). For this purpose dynamical synchronization between the frequency of auto-vibrations and the device taking electrical energy has been created.

The method and devices for conversion of vibrational motion (and energy) into rotational motion using dynamical synchronization has been created.

Sometimes synchronization is encountered when implementing the compatibility of external vibrations and waves with living organisms.

Some methods of dynamical desynchronization have been created, which are useful in some technological processes and in living nature.

Stabilization. The quality of some systems to self-organize their structures, trajectories and laws of motion, was called the effect of dynamical directivity (coauthor).

Some multidimensional systems have a quality to dynamically choose the trajectory or direction of their motion. This was called the effect of dynamical directivity. One can construct a great number of systems having such qualities. One example of such system is the generator of mechanical vibrations that is attached in such a way that it has supplementary degrees of freedom. Here some elementary examples are provided.

Case 1: system with a vibroexciter.

In Fig. 5 the system with two degrees of freedom consisting from two members 1 and 2 is shown. The case of the vibroexciter 3 is attached to the member 2 by a hinge. The member 4, exciting vibrations with mass \( m \), moves according to the line \( AB \) by a prescribed law. The member 1 may rotate by an angle \( \beta \) about the point \( O \), the member 2 can also move radially according to the line \( OA \). The case of the vibroexciter 3 may rotate by an angle \( \alpha \) with respect to the member 2. It is assumed that the member 2 has a concentrated mass \( m_2 \) at the point A. The member 2 is attached to the member 1 by a spring with the coefficient of stiffness \( C_\rho \), and the member 2 is attached to the immovable foundation by a spring with the coefficient of stiffness \( C_\beta \). The notation is introduced \( \rho = OA, \ z = AB \). The point A of the member 2 contacts with the immovable foundation by viscous friction, with the coefficient \( H_x \) according to the axis \( Ox \) and with the coefficient \( H_y \) according to the axis \( Oy \).
Differential equations are the following ones:

$$\Lambda = mz \left[ z(\ddot{\alpha} + \ddot{\beta}) + 2\dot{z}(\ddot{\alpha} + \ddot{\beta}) - \left[ \ddot{\rho} - 2\rho (\dot{\beta})^2 \right] \sin \alpha + (\rho \ddot{\beta} + 2\ddot{\rho}) \cos \alpha \right] = 0,$$

(33)

$$\left( m_2 + m \right) \rho (\ddot{\rho} + 2\ddot{\rho}) + mz \left[ z(\ddot{\alpha} + \ddot{\beta}) + 2\dot{z}(\ddot{\alpha} + \ddot{\beta}) \right] +$$

$$+m \left[ z\rho(\ddot{\alpha} + 2\ddot{\beta}) + (\dot{z}\rho + z\dot{\rho})(\ddot{\alpha} + 2\ddot{\beta}) + (\dot{z}\rho - z\dot{\rho})\ddot{\alpha} \right] \cos \alpha +$$

$$\left[ \dot{\ddot{z}}\rho - z\dddot{\rho} - z\rho(\ddot{\alpha} + 2\ddot{\beta})\dddot{\alpha} \right] \sin \alpha$$

(34)

$$+C_{\beta}(\beta - \ddot{\beta}) + 0.5 \left[ (H_x + H_y) + (-H_x + H_y) \cos 2\beta \right] \rho^2 \dddot{\beta} + 0.5 \rho \dddot{\rho}(-H_x + H_y) \sin 2\beta = 0,$$

$$\left( m_2 + m \right) \left[ \dddot{\rho} - \rho (\dot{\beta})^2 \right] + m \left[ \ddot{z} + z(\ddot{\alpha} + \ddot{\beta})^2 \right] \cos \alpha - \left[ z(\ddot{\alpha} + \ddot{\beta}) + 2\dot{z}(\ddot{\alpha} + \ddot{\beta}) \right] \sin \alpha +$$

$$+C_{\rho}(\rho - \ddot{\rho}) + 0.5 \left[ (H_x + H_y) + (H_x - H_y) \cos 2\beta \right] \ddot{\rho} + 0.5 \rho \dddot{\rho}(-H_x + H_y) \sin 2\beta = 0.$$

(35)

It is assumed:

$$z = Z \cos \omega t,$$

(36)

and the investigation is performed by dividing the motion into the slow and fast ones.

Case 1, when $$H_x = H_y = 0.$$ From the equations (33), (34), (35) for the case of stationary motion it is obtained:

$$\bar{\Lambda} = m\left(\frac{Z\omega^2}{4}\right)^2 \frac{p_\rho^2 - p_\beta^2}{p_\rho^2 - \omega^2} \frac{p_\beta^2 - p_\rho^2}{(p_\rho^2 - \omega^2)} \sin 2\alpha,$$

(37)

where:

$$p_\rho^2 = \frac{C_\rho}{m_2 + m}, \quad p_\beta^2 = \frac{C_\beta}{(m_2 + m)\bar{\rho}^2}.$$

(38)

When $$p_\rho < p_\beta$$ stable regimes are:

$$\bar{\alpha} = 0,$$ when $$\omega \in \left(0, p_\rho\right)$$ and $$\omega > p_\beta,$$

and

$$\bar{\alpha} = \frac{\pi}{2},$$ when $$\omega \in \left(p_\rho, p_\beta\right),$$

(39)

and the other regimes are unstable.

Case 2, when $$\alpha = \frac{\pi}{2}, \quad \rho = \text{const}, \quad H_x \neq H_y.$$

Differential equation of motion is the following one:

$$\ddot{\beta} + 0.5 \left[ h_x + h_y + (-h_x + h_y) \cos 2\beta \right] \dddot{\beta} = a \cos \omega t.$$

(41)

For small values of $$a$$ the stationary fast motion $$\ddot{\beta}$$ takes place with a frequency $$\omega$$ about any position $$\bar{\beta}.$$

For large values of $$a$$ steady state regimes take place about positions $$\bar{\beta} = 0, \frac{\pi}{2},$$ and

$$\bar{\beta} = \frac{\pi}{2}, \frac{3\pi}{2},$$ with small and large friction depending on $$a$$ (Fig. 5).

Case 3, when the system is with distributed parameters.
The analyzed system is a beam, to which the cases of the exciters of vibrations that were analyzed earlier are attached by hinges. Differential equations of motion are:

\[
\frac{EF}{\partial^2 x^2} \frac{\partial^2 u}{\partial^2 x^2} + \zeta \frac{\partial u}{\partial t} - \rho F \left( \frac{\partial u}{\partial t} + \bar{u}_0 \right) = \sum_{j} \delta \left( x - x_j \right) F_{in} \left( u_j \right) \cos \alpha_j, \tag{42}
\]

\[
\frac{EJ}{\partial^4 x^4} \frac{\partial^4 v}{\partial^4 x^4} + \zeta \frac{\partial v}{\partial t} + \rho F \frac{\partial v}{\partial t} = \sum_{j} \delta \left( x - x_j \right) F_{in} \left( v_j \right) \sin \alpha_j, \tag{43}
\]

\[
F_{in} \left( u_j \right) = \left( m_0 + m \right) \left( \bar{x}_0 + \bar{u}_j \right) + m \left\{ \left[ \bar{z}_j z_j \left( \bar{\alpha}_j \right)^2 \right] \cos \alpha_j - \left( z_j \bar{\alpha}_j + 2 \bar{z}_j \bar{\alpha}_j \right) \sin \alpha_j \right\}, \tag{44}
\]

\[
F_{in} \left( v_j \right) = \left( m_0 + m \right) \bar{v}_j + m \left\{ \left[ \bar{z}_j z_j \left( \bar{\alpha}_j \right)^2 \right] \sin \alpha_j + \left( z_j \bar{\alpha}_j + 2 \bar{z}_j \bar{\alpha}_j \right) \cos \alpha_j \right\}, \tag{45}
\]

\[
M_{in} \left( \alpha_j \right) = J \ddot{\alpha}_j + m z_j \left[ \bar{z}_j z_j \bar{\alpha}_j + z_j \bar{\alpha}_j - \left( \bar{x}_0 + \bar{u}_j \right) \sin \alpha_j + \bar{v}_j \cos \alpha_j \right] + H \bar{\alpha}_j, \tag{46}
\]

\[
c x_0 = \frac{EF}{\partial^2 x^2}, \quad \frac{EJ}{\partial^3 x^3} \frac{\partial^2 v}{\partial^2 x^2} = 0, \quad \frac{EJ}{\partial^2 x^2} = 0 \text{ at } x = 0. \tag{47}
\]

Analytical calculations are performed in the same way as for the systems with concentrated parameters. The values \( \alpha = 0 \) and \( \alpha = \frac{\pi}{2} \) are obtained and this means that in the system transverse or longitudinal waves exist.

Further analysis is performed by numerical methods.

It is to be noted that in the systems when the frequency of external excitation is small, much smaller than the first eigenfrequency, the system performs vibrations with a deterministic frequency. For the case when the frequency of excitation is near to the first eigenfrequency or greater than it, especially for bigger amplitudes of excitation, motions of chaotic type arise. On this basis generators of mechanical vibrations of chaotic type are constructed which are required in a number of vibrational technological processes and also for excitation of biological systems.

The effect of dynamical directivity itself is useful in some technological processes and engineering.

Vibrations and waves are an advanced means for stabilization of some systems. Unstable regimes may be converted into stable ones and stable regimes can be converted into unstable ones, or a higher or lower number of stable and unstable regimes can be created, or regimes of quite different type can be created. This is illustrated by the simplest example – the pendulum hanging on a hinge. In the case of statics the stable position of the pendulum is when its mass in the vertical line is located in the lower position, and in the upper position it is unstable. Other authors showed that by performing straight linear vibrations by harmonic law vertically of the point of hanging of the pendulum, the stable and unstable positions may interchange, besides two stable and two unstable positions may exist. Or because of vibrations the pendulum may perform stable rotations at combined frequency with the frequency of excitation. But if the gravitational force does not act into the pendulum, then the pendulum may be located in any position and it will be in a stable status. By vibrating the foundation of the pendulum in its plane it will take a parallel position with respect to the direction of vibrations. This quality is useful in some technologies.

**Stabilization of synthetic fiber.** Here in the process of production viscous fluid flows vertically downwards from the filler and in the process of motion it cools and hardens into a thread. It has been recommended to vibrate the liquid mass before hardening by a magnetic method at Megahertz frequencies. It occurred that the domains because of vibrations located...
themselves in an organized way. It was determined that the threads obtained due to longitudinal and transverse vibrations have very big differences in their resistance in Ohms, also their difference in strength was obtained.

Here there are perspectives to clean and structurize water and other fluids by using mechanical and electrical vibrations. This fact is interesting when applying vibrations to stabilize or destabilize the systems of blood flow, lymph, nervous activity.

The method to perform an opposite work by using vibrations and waves, that is to destructurize and destroy molecules or their connections, was proposed. The investigations were started with the purpose of destroying molecules of water by using resonance excitations in order to divide the molecules into hydrogen and oxygen.

Stabilization by air cushions (coauthor) has been used for stabilization of bodies, at the same time reducing the forces of friction and increasing sensitivity, minimizing harmful vibrations. A heavy rotor located on two aerostatic bearings with air supply and distribution of special form minimizes radial and axial vibrations of the rotor. Also a system with a body performing translational motion in one direction has been created, which with the help of air cushions centers the axis of motion, minimizes transverse vibrations.

Similar systems which are based on travelling waves have been developed.

An active system of stabilization of rotors has been proposed (coauthor). Each of the bearings of the rotor of fast motion with two supports is attached to the foundation by orthogonal springs. Electromechanical vibrators are connected in parallel with the springs. The converters measure vibrations and automatically controlled vibrators minimize harmful vibrations in the region from tens up to several hundreds of Hertz. A similar system has been created for active protection of human hearing elements from harmful external noises. Also a system for suppression of macro-vibrations with the help of micro-vibrations of high frequency has been created.

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Vibrostabilizers of motion (coauthor). Study of rigid and flexible body vibrostabilizers of motion developed on the basis of vibrational motion transducers, vibrational supports and vibrational guides is overview.

On the basis of the survey of currently existing vibrostabilizers new vibrostabilizers of motion in the form of vibrosupports were created and means of control of their friction characteristics were proposed.

Methods of finite element modeling of contact pairs of links of vibrostabilizers of motion were investigated. Automatic finite element discretization of links is performed. The equations of dynamics of links and the matrices of contact geometry and mechanics were set up. On their basis dynamic equations of the link were transformed and control of dynamics of the vibrostabilizer at various phases of its motion was obtained. Algorithms for integrating the equations of motion of the vibrostabilizer and for defining kinematic parameters, such as displacement, velocity and acceleration, were developed. Algorithms for dynamic analysis of vibrostabilizers and for the definition of dynamic characteristics of their operation, forces of contact interaction and their energy were developed also.

Theoretical studies of the vibrostabilization forces of friction and dynamic characteristics of the movable units in sliding and rolling bearings were performed. In this process their dynamic models were developed and differential equations of motion of the body along vibrating surfaces were set up. The latter were analyzed for particular cases by numerical methods. Graphical relationships were obtained and investigated.

Analysis of mechanical interaction between a cylindrical vibrostabilizer and a flexible tape parametrically was performed by a numerical method. Finite element models of a tape and a vibrostabilizer and their contact interaction was investigated, taking into account longitudinal and transverse rigidity of tape contact effects between the tape and the vibrostabilizer under vibro-impact conditions incorporating nonlinearity of the vibrostabilizer. Rational conditions
for mechanical interaction between the tape and the vibrostabilizer were suggested and the expediency of application of parametric resonance for increasing the amplitude of vibrations of a surface guide was determined. The data of experimental investigations were analyzed. Influence of the vibrostabilizer upon the accuracy of tape motion in a tape drive mechanism was established.

Data of theoretical and experimental analysis of formation of clearance and its stabilization between the operating surface of a vibrostabilizer and a flexible tape advanced under it were investigated. For this purpose dynamic models were developed and differential equations were obtained. They were solved for a particular case by the method of finite differences. A new procedure and experimental facilities were implemented. The experimental data relevant to the interaction of a vibrostabilizer and a magnetic tape, its contact being in close proximity to an operating clearance of a magnetic head, were investigated.

2. Nonlinear effects of vibrations and waves in the creation of mechanical systems, machines, robots

**Transportation by waves. Vibromotors.** Case 1: travelling waves.

The system with one degree of freedom is presented, in which the travelling transverse and longitudinal waves excited in one body give stepping linear motion to the output system by the contact way (Fig. 6).

![Fig. 6. Wave motor: 1 – profile of the input member, 2 – the output member and its 3 – case of the directing part, 4 – pressing spring, 5 – immovable directing parts](image)

The waves are described by the following equations:

\[ \eta \left[ \omega t - k \left( x - \eta \right) \right], \]
\[ \xi \left[ \omega t - k \left( x - \alpha \eta \right) \right], \tag{48} \]

where the longitudinal \( \eta \) and transverse \( \xi \) waves are periodic functions of their arguments,

\[ k = \frac{2\pi}{\lambda}, \quad \lambda \text{ is the wavelength}, \]
\[ \alpha \in [0, 1]. \tag{49} \]

In the equations by assuming \( \eta = \varepsilon \eta \), where \( \varepsilon \) is a small parameter, and expanding into a power series:
\[ \eta = \eta \left[ \omega t - k (x - \varepsilon \eta) \right] = \eta_0 + \varepsilon k \eta_0 \eta' + \varepsilon^2 \ldots, \]
\[ \xi = \xi \left[ \omega t - k (x - \varepsilon \xi) \right] = \xi_0 + \varepsilon \alpha k \eta_0 \xi' + \varepsilon^2 \ldots, \quad (50) \]
where:
\[ \eta_0 = \eta (\omega t - k x), \]
\[ \xi_0 = \xi (\omega t - k x), \quad ' = d/d (\omega - k x). \quad (51) \]

In the case when contact transportation takes place it is obtained:
\[ \dot{y} = (\omega - k \dot{x}) \xi' + \omega k \eta \xi', \]
\[ \ddot{y} = -k \dot{x} \xi'' + (\omega - k \dot{x}) \xi'' + (\omega - k \dot{x}) k (\eta' \xi''). \quad (52) \]

By assuming dry and viscous friction at the point of contact the following differential equation of motion is obtained:
\[ P_x - k \xi' P_y + (k \xi' P_x + P_y) f_0 \text{sgn} \dot{s}_{12r-t} + f_1 \dot{s}_{12r-t} = 0, \quad (53) \]
where:
\[ P_x = (m_0 + m) \ddot{x} + F_x, \quad P_y = m \ddot{y} + C (\xi + b), \quad (54) \]

\( f_0 \) and \( f_1 \) are the coefficients of dry and viscous friction,
\( F_x \) is the force of resistance,
\( C \) is the coefficient of stiffness of the spring and \( b \) is its fastening.

At the point of contact the angle of the tangent line \( \gamma \) is determined by the following equation:
\[ \tan \gamma = -k \xi', \quad (55) \]
and the velocity of slippage:
\[ \dot{s}_{12r-t} = \left( 1 + k^2 \xi'^2 \right) (\ddot{x} - \omega \eta') \cos \gamma. \quad (56) \]

The equation (53), by taking into account (54 – 56), takes the following form:
\[ (1 + \mu) \ddot{x} + f_x - k \xi' \left[ \ddot{y} + p^2 (\xi + b) \right] + \left[ k \xi' \left[ (1 + \mu) \ddot{x} + f_x \right] + \ddot{y} + p^2 (\xi + b) \right]. \quad \cdot f_0 \text{sgn} (\ddot{x} - \omega \eta') + h \left( 1 + k^2 \xi'^2 \right) (\ddot{x} - \omega \eta') = 0, \quad (57) \]
where:
\[ \mu = \frac{m_0}{m}, \quad f_x = \frac{F_x}{m}, \quad p^2 = \frac{C}{m}, \quad h = \frac{f_1}{m}, \]
\[ \eta' = \eta_0' + \varepsilon k \left( \eta_0 \eta'_0 \right)' + \varepsilon^2 \ldots, \quad \xi' = \xi_0' + \varepsilon \alpha k \left( \eta_0 \xi'_0 \right)' + \varepsilon^2 \ldots. \quad (58) \]

Analytical computer investigation of the equation enables to obtain final analytical relationships of the stationary regime and in its vicinity and also to reveal the nonlinear effects of the system.

The case when:
\[ \xi = f_0 = 0, \quad \eta = A \cos \left[ \omega t - k (x - \eta) \right]. \quad (59) \]

By using according to the equations (50), (51), (56) the linear part of the series of the expansion of \( \eta \) with respect to \( \xi \) it is obtained:
\[ \ddot{x} + h \left[ \ddot{x} - \omega \left[ \eta_0' + k (\eta_0 \eta'_0) \right] \right] = 0, \quad (60) \]
\[ \eta_0 = A \cos (\omega t - k x). \quad (61) \]
In the equation (60) the substitution is performed:
\[ x = vt + \bar{x} + \bar{x}, \]  
where \( v \) is the velocity of the steady state regime,
\( \bar{x} \) and \( \bar{x} \) represent the slow and fast motion respectively.

The differential equation of the fast motion is:
\[ \ddot{x} + h\omega A \sin \left( (\omega - kv)t - k\bar{x} \right) - h\omega kA^{2} \cos 2 \left( (\omega - kv)t - k\bar{x} \right), \]  
and the differential equation of slow motion is:
\[ \ddot{x} + h\omega A \sin \left( (\omega - kv)t - k(\bar{x} + \bar{x}) \right) - h\omega kA^{2} \cos 2 \left( (\omega - kv)t - k(\bar{x} + \bar{x}) \right). \]  

Usually the motion takes place slowly when compared with the frequency of excitation, that is:
\[ v << \frac{\omega}{k}, \]
thus it is possible to assume:
\[ \omega - kv \approx \omega, \]
and from the equations (63), (64) the value of average velocity is obtained:
\[ v = 0.5h^{2} \omega^{-1}kA^{2} \left( 1 + 0.25k^{2}A^{2} \right). \]

Case 2: standing waves.
The system is presented in which the transverse and longitudinal standing waves excited in one body provide stepping linear motion to the output member by the contact way (Fig. 6).

The waves are described by the following equations:
\[ \eta = \eta_{1}\left[ k\left( x - \eta \right) \right] \eta_{2}(\omega t), \]
\[ \xi = \xi_{1}\left[ k\left( x - \alpha \eta \right) \right] \xi_{2}(\omega t), \]  
where \( \eta_{1}, \xi_{1}, \eta_{2}, \xi_{2} \) are periodic functions of their arguments.

In the same way as in the equations (50) \( \eta_{1}\left[ k\left( x - \eta \right) \right] \) and \( \xi_{1}\left[ k\left( x - \alpha \eta \right) \right] \) are expanded into the power series:
\[ \eta_{1}\left[ k\left( x - \eta \right) \right] = \eta_{10} + \varepsilon k\eta_{10}\eta_{10} + \varepsilon^{2} \ldots, \]
\[ \xi_{1}\left[ k\left( x - \alpha \eta \right) \right] = \xi_{10} + \varepsilon k\eta_{10}\xi_{10} + \varepsilon^{2} \ldots, \]  
where:
\[ \eta_{10} = \eta(kx), \]
\[ \xi_{10} = \xi(kx), \quad ' = d/d(kx). \]

In the case of contact transportation:
\[ \dot{y} = k\xi_{1}' \xi_{2}' \left( \dot{x} - \omega \eta_{1}' \right) + \omega \xi_{1}' \xi_{2}', \]
\[ \ddot{y} = k\xi_{1}' \xi_{2}' \dot{x} + k^{2} \xi_{1}'' \xi_{2}'' \dot{x}^{2} + \left( 2k\xi_{1}' \xi_{2}' \xi_{1}'' \xi_{2}'' \eta_{1}' \eta_{2}' - k^{2} \xi_{1}'' \xi_{2}'' \eta_{1}' \eta_{2}' - k^{2} \xi_{1}' \xi_{2}'' \eta_{1}' \eta_{2}' \right) \omega \dot{x} + \left( \xi_{1}'' \xi_{2}'' - k\xi_{1}' \xi_{2}' \eta_{1} \eta_{2}' \right) \omega^{2}, \]  
where:
\[ \eta_{1}' = \eta_{10}' + \varepsilon k\left( \eta_{10}' \eta_{10} \right) + \varepsilon^{2} \ldots, \]
\[ \xi_{1}' = \xi_{10}' + \varepsilon k\left( \eta_{10}' \xi_{10} \right) + \varepsilon^{2} \ldots, \]  

\[ \eta_2' = d\eta_2/d(\omega t), \quad \xi_2' = d\xi_2/d(\omega t). \]

At the contact point the angle of the tangent line \( \gamma \) is determined by the equation:

\[ \tan \gamma = k_1 \xi_1' \xi_2, \]

and the velocity of slippage:

\[ \dot{s}_{12i} = \left[ 1 + \left( k_1 \xi_1' \xi_2 \right)^2 \right] \left( \dot{x} - \omega \eta_1 \eta_2' \right) \cos \gamma. \]

In the same way as (53), (54), (57), (58) by taking into account (68 – 73) differential equation of motion is obtained in the following form:

\[
(1 + \mu) \ddot{x} + f_x + k_1 \xi_1' \xi_2 \left[ \ddot{y} + p^2 (\xi + b) + \left( (1 + \mu) \ddot{x} + f_x \right) / \omega + p^2 (\xi + b) \right] + h \left[ 1 + \left( k_1 \xi_1' \xi_2 \right)^2 \right] \left( \dot{x} - \omega \eta_1 \eta_2' \right) = 0.
\]

In the case when:

\[ \omega >> k_1, \]

the differential equation (74), by taking into account (75) and \( b = f_0 = 0 \), takes the following much simpler form:

\[
(1 + \mu) \ddot{x} + k_1^2 \xi_1' \xi_2 \left[ \ddot{y} + p^2 \xi + b \right] + h \left[ 1 + k_1^2 \xi_1' \xi_2^2 \right] \left( \dot{x} - \omega \eta_1 \eta_2' \right) = 0.
\]

The case when:

\[ \eta_1 = A \cos k (x - \eta), \quad \eta_2 = \cos \omega t. \]

By the method of division of motion into the slow and fast one it was determined that the dynamical equilibrium of the system is located in the nodal points.

The case when:

\[ \xi_1 = B \sin k (x - a \eta), \quad \xi_2 = \sin \omega t. \]

In the same way it is determined that:

when \( p^2 - \omega^2 > 0 \) the stable positions are in the nodal points of the wave,

and when \( p^2 - \omega^2 < 0 \) the stable positions are in the maximum points of the extreme deflections of the wave.

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Vibromotors for precision microrobots (coauthor). Considerable progress has been achieved in all branches of technical sciences and technology, however especially noticeable changes have taken place in instrument construction. In precision instrument development the advances of precision mechanics, optics, electrotechnics and electronics (as well as automatic and remote control) have been used. Specifications for various types of devices become more demanding: they should be small, reliable, have high responsiveness and high efficiency and be able to operate in extreme conditions (in vacuum, under acceleration and vibratory overloads, in a wide temperature range and at increased level of radiation). Especially strict requirements have been imposed for actuation elements and drives of precision devices and instruments.

Electromotors do not always satisfy the increased demands: they have a large time constant, introduce elements of low rigidity into the dynamic system and have a limited range of speed. The resolution of electric motors (including stepper motors and motors with reducers) is relatively low and is measured in units of angular minutes or tenths of a millimeter in the linear drives. Such characteristics cannot satisfy the increased demands of electromechanical units. Therefore the development of a new type of drive, based on the transformation of high
frequency micro-vibrations into directed motion, is of great interest to designers of precision devices.

By analogy with electrical, pneumatic and hydraulic motors it has been suggested to call high frequency vibration motion transformers as vibratory motors or vibromotors.

In the first designs of vibromotors oblique impacts typical for low frequency vibratory conveyance were used. Further research and development led to the application of finer effects of interaction between vibrating members. Wave motion vibromotors and vibromotors with controllable liquid links (in the form of electrorheological and magnetorheological liquids) were developed. Compressible air films were also used in vibromotors.

The first studies of vibromotors have shown their exceptional properties: high resolution of displacement, wide temperature range and the absence of parasitic magnetic or electrical (in particular radiation) fields. Vibromotors can be designed without metal parts (besides power supply cables), thus allowing application in precision magnetic and geodetic instruments. But the main feature of vibromotors is their dynamic quality in conditions of transient motion (i.e. in start – stop and stepping regimes), because the vibrating element becomes a brake at the moment of switching off of the power supply.

Application of new materials of high piezoactivity and mechanical materials and alloys with special properties (basically for the contact materials) was of great significance for vibromotors. The design was simplified by applying new measuring and diagnostic methods, such as holographic interferometry and acoustic emission diagnostics.

A new class of automated devices has been developed – robot manipulators. Robots of the first generation have been implemented, robots of the second generation have been developed and the possibilities of designing robots of a third generation have been studied. Vibromotors are of great importance in designing precision microrobots – devices capable of manipulating objects of small mass in a limited space but with very high accuracy.

The application of such microrobots is very broad – from microsurgery to assembly of complex microelectronic circuits. The application of vibromotors with several degrees of freedom enables the design of micromanipulators of ultra-high accuracy.

In many fields of application vibromotors solve the problems of positioning and uniform high speed displacements and execute prescribed motions. Mechanisms with controllable structure and mechanisms with controllable parameters are also very effective.

To expedite the implementation of vibromotors many design diagrams providing various functional possibilities have been obtained.

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Capillary effects have been used for the development of some systems. 

Case 1 – capillary mechanisms of micro-displacements.

For example the wooden parts of some types of trees are relatively homogeneous and have very small capillaries. By using high frequency vibrations it is possible to fill the capillaries with fluid in them and to remove the fluid from the capillaries. Thus the material with capillaries in the process of expansion and contraction performs micro-motions. From such mechanisms robots of micro-displacements with any desirable number of degrees of freedom may be produced.

Besides, the micro-capillaries have useful application for micro-dosing of fluids.

Case 2 – capillary restoring force.

This effect may arise in micro-porous materials or structures of ply type or between the two bodies with areas of contact between those two bodies. After filling those materials with fluid and after that compressing them areas between separate parts occur where there is no more fluid. After the reduction or removal of the pressing force the fluid immediately flows into the areas, from which it was pushed away. Thus the impulse restoring force is induced. This has been called the capillary restoring force. The process of compression is shown in Fig. 7, where part of the graphical relationship $BC$ is the graphical representation of the capillary restoring
force as a function of displacement. Thus this effect is useful in some technological processes, also in generating stepped displacements of bodies in space.

Fig. 7. Force – deformation characteristic

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**Design of mechanical systems with high kinematic pairs.** When the laws of transformation of motions and energies between separate members of the system are prescribed there is a possibility to freely choose in definite regions the dissipation of energy, dimensions, masses of separate elements or of the whole system. Here a possibility occurs to optimize the systems according to the mini-max criteria.

The method of design of systems according to its mini-max distance from the self-stopping conditions by taking into account the dissipative forces in high kinematic pairs has been created. Graphical and analytical methods of design of mechanical systems of several types were elaborated.

In the systems of force type it is important to ensure mini-max wear of high kinematic pairs. For this purpose an analytical method which determines the intensity of wear according to the normals of the contacting surfaces was created. For this purpose analytical determination of differentials of slippage and rolling of contacting profiles and of other parameters was used. The method for the design of mechanisms of this type according to the mini-max wears was created. This was later experimentally confirmed by other authors.

The development of the method of design of frictional mechanical systems with one and with two curvilinear profiles with variable coefficient of transfer of motion between the input and the output members was performed. High-frequency micro-vibrations that were used expand the possibilities for application of those systems. Here the nonlinear differential equations for the synthesis of profiles are taken into account.

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**Dynamics of mechanisms of rolamite type (coauthor).** Problems of mechanisms of rolamite type (MRT), their development and experimental research of dynamics are investigated. MRT are used in precision machine tool construction, in gauges and measurement instruments, in medicine, in chemical and food processing industries, in robots and space exploration devices.

Technical solutions, analysis of development, specifications and classification of the MRT are investigated.

The density criterions of the MRT structure were determined. Theoretical studies of structural links, methods of calculating kinematic precision of mechanisms, studies of their dynamic non-stability were performed. The above mentioned experiments have been analyzed. New constructions of high precision and advanced performance were proposed and introduced.

The results of theoretical studies and experiments of the MRT with vibrating elements were analyzed, some designs of new MRT were proposed and fields of their application were pointed out.
Methods and computer software for calculation of small elastic oscillations of the MRT elements, based on experimental data of holographic interferometry and the theory of oscillations of mechanical systems were investigated.

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**Dynamic synthesis of machines by semi-physical modeling (coauthor).** Basic principles of dynamic synthesis of functional parts of machines by semi-physical modeling were investigated.

It was demonstrated that it is possible to choose the most suitable constructions or machine couplings by semi-physical (semi-empirical) modeling if the whole machine, with the exception of the analyzed part, is presented formally; and also it is possible to perform synthesis of a functional part of the machine, while the other functional parts are presented as real, that is in constructional implementation. Earlier semi-physical models have not been used for dynamic synthesis because of the lack of theoretical investigations.

Exit chain division of mechanisms into real and formalized parts was analyzed, that are changed into half-physical models further on.

The principles of derivation of differential equations of semi-physical models of machines with linear and non-linear functional parts and with different couplings were worked out. On this basis stability research of semi-physical models, including machines with non-holonomic couplings, was performed. Analytic and graphic equations defining stability regions were obtained.

Main requirements for actuating mechanisms of semi-physical models were determined. Analytical research of the dynamics of such mechanisms was carried out, optimal motion laws of their chains were calculated. On the basis of the analytical research and stability requirements of semi-physical models new actuating mechanisms were constructed, this enabled the connection of real and formalized machines, of their parts into a stable semi-physical model.

Principles of selection of structure of formalized parts of semi-physical models were analyzed, analogical models of the formalized functional machine parts containing electromagnetic mechanisms, gaps between chains, and including complicated physical phenomena of chain interaction were defined.

Main assumptions of choosing formalized parts of machines by semi-physical modeling were investigated. Results of the analysis of building up preliminary structures of formalized machine parts were obtained. It was demonstrated that definition of a preliminary structure of formalized parts allows to reduce the time of semi-physical modeling and of obtaining the result of synthesis.

Experimental setups were constructed on the basis of the performed research. Results of synthesis of mechanical parts of tape drive mechanisms, vibro-isolation systems with an operator and rotary machines, also of synthesis of drives of measuring and grinding machines were obtained. The main principles for defining the way of synthesis were determined. It is revealed that theoretically it is possible to prove the necessity of semi-physical modeling for the synthesis of specific classes of machines.

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**Dynamics of precise tape drive mechanisms (coauthor).** Investigations of the dynamics of precise tape drive mechanisms have been generalized and the recommendations concerning the optimal construction of the mechanisms applied in precise magnetic recording and in recorders of dynamic processes of diverse mechanisms and their sets have been prepared.

Dynamic models of tape drives were represented as multi-degree of freedom dynamic systems and specified by the system of differential equations in the form of matrix convenient for computer-aided design.

Dynamics of basic units of tape drives were investigated. The previously unknown technique of system of uncombined loops moving in the accumulator has been proposed by taking into consideration transient conditions of motion. Methods of compilation of the dynamic
model of a rotating unit enveloped by a flexible tape have been established. Possibility of the slip of the tape on the rotating unit has been taken into account. As a result of the solution of differential equations of the system optimal working conditions of rotating units in tape drives have been developed for the first time.

Synthesis of tape drives according to the frequency spectrum on the basis of matrix algebra has been proposed. Peculiarities of the frequency spectrum synthesis as of an un-stationary dynamic system have been considered.

Frequency spectrum synthesis is also accomplished on the basis of the topological models expressed by the conservative, dissipative and pulsed systems, as well as systems having deviating argument and distributed parameters.

Principles of dynamic diagnostics of tape drives were proposed. Signals bearing diagnostic information have been determined. Diagnostic parameters have been chosen and accuracy of their determination has been estimated. Some informative signal forming methods and facilities have been specified. Problems of extraction of diagnostic information have been investigated.

Recording, reproduction and recording – reproduction error analysis for the case when a tape drive is subjected to random disturbances, the latter representing multidimensional random processes, was performed. Analysis of ordinary probability properties of the mentioned errors, such as the mean value and dispersion, has been performed. Asymptotic distribution of errors when the registration time is unlimited has been determined and the rate of their convergence and dissociation has been investigated.

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**Precise vibromechanic scanning devices (coauthor).** Precise laser emission scanning devices developed on the basis of the principles and by means of vibration mechanics are of high interest in engineering.

Original schemes and designs of a new subclass of scanning devices possessing a piezoelectric vibrodrive whose number of degrees of freedom of the element being scanned ranges from 1 to 4 were proposed. The operation of these devices has been considerably improved due to the introduction of additional degrees of freedom. The design and the principles of operation of those devices were investigated. The scheme for constructing a scanning device with a spin axis of a scanned element to be controlled in space was proposed. Profound research has led to the design procedure of scanning devices and the synthesis of exciting zones of a multidimensional vibration transducer.

Schemes and designs of scanning devices having bimorph piezoelectric drive were proposed. Their basic characteristics and design procedure were thoroughly investigated. The results of simulation of piezodrives of various geometric shapes having complex profile of control electrodes were obtained.

Solid state scanning devices manufactured on the principle of thin film technology were investigated. These devices distinguish themselves by their small dimensions and quick operation. By applying bulk technology the elements of a scanning device and those of an electronic circuit of its control have been overlapped on a silicon chip. In this way the integration of mechanical and electronic elements has been accomplished, thus opening new ways for their application.

Manufacture of scanning devices with changeable surface reflecting geometry was investigated. The structures of these devices and design procedures for the determination of the values of their parameters were proposed.

Original precise scanning devices can be effectively used in the production of sensitive optical – mechanical – electronic instruments, laser techniques, engineering and microelectronics.

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**Systems based on constant magnets.** Constant magnet motor or actuator consists from the input member 1 and the output member 2 (Fig. 8).
Two mutually parallel rows of constant magnets (or other sources of energy) 3 and 4, and also the wave (curvilinear) profile 5, are attached to the input member 1. The magnets in the rows 3 and 4 are located by opposite poles one against another with constant step. The wave profile is a periodic function of displacement parallel to the lines of location of the magnets 3 and 4. The output member 2 is kinematically connected to the wave profile 5, which may move with respect to the input member parallel and perpendicular to the directions of location of magnets 3 and 4. Constant magnets are attached to the output member 2 parallel to the rows of magnets 3 and 4. In all the parallel rows 3, 4 and 2 magnets with equal distances between poles are attached. All magnets are of the same power. The position of equilibrium of the output member 2, \( y = 0, \quad x = 0 \), is ensured by the elastic – dissipative member 6.

On the basis of constant magnets generators of mechanical vibrations with one and two coordinates have been created with co-authors. They can be noted for having comparatively small weight than the known electro-magnetic and electro-dynamic ones. The schematic representation of one of them is presented in Fig. 9. The system consists of the input member 1 and the output member 2. The output member 2 is attached to the immovable foundation with the elastic elements 3 and 4, which allow the output member to perform vibrational motion according to the rotational and translational coordinates. Constant magnets are attached to the input and output members of disc shape. Motor provides rotational motion to the input member 1. When the input member performs rotational motion the forces of interacting magnets provide two dimensional vibrations to the output member.

**Vibration of bearings (coauthor).** Continuous progress of some significant branches in engineering (manufacture of flight vehicles, instruments and machine tools, automotive, electric machine engineering and other industries) is linked with the use of numerous rolling contact
and sliding bearings. In many cases performance of the instruments and devices is highly dependent on dynamic phenomena taking place in bearings. That is why the questions of technical diagnosis of the whole unit are so inseparably related to the diagnosis of the bearing assembly, which by itself represents an autonomous rotor type system which is the basic source of all undesirable disturbances in the machine or instrument as a whole. This can be evidently seen in gyroscopic instruments in which the friction torque in the bearings causes appreciable deterioration of the dynamic characteristics of the whole setup due to vibrations. In tape recorders oscillations of the speed of tape movement measured by the coefficient of detonation are caused by defects in the geometric sizes, accuracy of rotation and variation in friction torques in the ball bearings used as the rolling contact support of all the rotating components (tone shaft, guide rollers, etc.).

Extensive studies aimed at determining dynamic characteristics of the bearings and rotor systems working under different conditions have been carried out.

All phenomena taking place in the bearing assemblies are to greater or lesser extent related to vibrations. Consequently a change in the friction torque, varying parameters of a lubricating film in bearings, oscillations of rings and other processes may be treated as vibrational changes. Methods of analysis and data processing for these processes are identical and thus all the processes are considered under the common headline of vibratory processes.

In order to make diagnostic ability more efficient the processes which take place in bearings and bearing assemblies have been analyzed.

The tasks posed to industry are successfully solved by improving the quality and expanding the capabilities of technical means for the control and measurement of parameters of bearings and rotor systems.

Registration of the parameters however has only an auxiliary character, because the basic goal of measurements is to facilitate the diagnosis of the object being tested by taking into account its dynamic characteristics.

Ball bearing supports and bearing assemblies may be diagnosed on the basis of various characteristics. One of the most reliable techniques is the diagnosis of bearings and their assemblies using the following parameters: friction torque, hydrodynamic oil film, vibratory characteristics and temperature levels of the interacting components in the bearing assemblies. The interrelation between the state of the lubricating layer and working capacity of the bearing assembly has been investigated.

All the investigations that have been performed were aimed at determination of the technical state of the bearings and their assemblies and assurance of their further safe operation in a given assembly or machine.

The factors affecting the working characteristics of bearings have been analyzed and the basic techniques and means of measurement for the processes taking place in bearings have been proposed. A methodology for an analytical study of characteristics of bearings has been proposed and numerous results of experimental investigations dealing with rolling contact bearings have been obtained.

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Vibrations of rotor systems (coauthor). Theoretical and experimental investigations of vibrations of precise rotor systems have been performed.

General questions of vibrations of rotor systems have been investigated, which include analysis of basic reasons of vibrations and some methods and devices for avoiding of vibrations.

A number of problems related with balancing of rotor systems for the case of stiff rotor have been investigated. In this case two qualitatively different vibrating systems are analyzed, which are characterized by plane – parallel and spatial vibrations.

Problems of determination of parameters of non-axiality in the process of centering of a connection of two rotor machines have been investigated. Peculiarities of dynamical centering of a connection of two rotor machines, the rotors of which are connected by an elastic clutch,
relationship between the parameters of non-axiality and the parameters of vibrations, conditions of choice of their optimal values and other problems have been investigated.

Data on the choice of some structural parameters of rotor systems on the basis of the conditions of stability have been obtained.

Investigations of vibrations of rotor systems by statistical methods have been performed. Relationships for the optimal estimation of correlation functions and spectral densities of realizations when performing calculations by a computer have been determined.

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**Thermomotors (coauthor).** Original methods and means for conversion of heat energy into the energy of forced one-dimensional or two-dimensional motion of a body have been proposed. This is being implemented with the help of a rigid deformable body. Here heat sources of various types are used (of hot fluid or gas, of rays of the sun) and for the taking of mechanical energy the motion of a rotor is used, which rotates the propeller, or the screw of a ship, or a dynamic machine or other type of working member. The excessive heat energy is also taken for various purposes. Such drives are in some cases suitable for excitation of vibrations and waves.

One of the schematic representations is illustrated in Fig. 10. The system consists of the input member 1, the output member 2, the heat accepting member 3, the member for removing the mechanical energy 4 and of the member for removing excessive heat 5. Because of thermal deformation the input member 1 moves and rotates the output member 2.

![Fig. 10. Thermomotor](image)

**Multi-functional pneumatic mechanisms operating on the air cushion (coauthor).** The principle of operation of mechanisms of this type is based on the unstable flow of compressed air through cavities of definite configuration. On the basis of this principle pneumatic vibro-converters of one or several coordinates operating in the regime of auto-vibrations have been created. The described principle has been used in the development of rotational drives as well as the drives for transportation of the tape of paper, of the taking member of a robot for performing of automatic assembly of parts of “screw – shaft” type. For this purpose the robotic device has been created which can operate in three regimes, that is in the contact regime when the part is being taken and carried to the position of assembly; in the air cushion regime when orientation of the part is performed; and the third vibrational regime of operation when the assembly of the “screw – shaft” takes place (this regime is used when there is a small non-coincidence of geometrical axes of the assembled parts). In recent years pneumatic drive of a robot which may be applied for the performance of non-destructive diagnostics and identification of internal surfaces of pipe lines has been created (Fig. 11).

Experimental and numerical investigations of the created multi-functional vibro-converters have been performed. The motion of the mechanical system was described by several systems of non-linear equations by using the Reynolds, Saint-Venant, Vantsel and Prandtl equations of air flow. The regions of existence of auto-vibrations and of the air cushion have been obtained, graphical and theoretical relationships of characteristics of auto-vibrations from the geometrical and dynamical parameters of the vibro-converter have been determined. The obtained results of
investigations enable to apply such vibro-converters in various technological processes and also in the robotized adaptive mechatronic systems.

**Fig. 11.** Part of the schematic representation of the pneumatic drive of a robot: 1 – case of the drive of a robot, 2 – a cylindrical glass of the vibro-converter, 3 – a spring, 4 – a flexible needle, 5 – support of the operating member, 6 – member for supplying of pressure $P_1$, 7 – the pocket of the vibro-converter, 8 – the place from which material is taken out, 9 – the taking element of the working member, 10 – the working gap of the air cushion, $P_1$ – the supplied pressure, $\alpha_1$ – the angle of inclination of fastening of the needle, $r_1$ – radius of the member for supplying of pressure, $r_k$ – internal radius of the glass, $r_a$ – external radius of the glass, $l_k$ – the height of the pocket, $m$ – mass of the working member 5, $m_1$ – mass of the glass

### 3. Vibration control

Methods and means for measurement of vibrodisplacements of bodies have been proposed in cases when disturbances act into the measurements, such as the unevenness of the surface of the body, deformations of the body itself, vibrations according to other coordinates than the measured ones. Measurement of vibrations of bodies with several degrees of freedom by eliminating disturbances has been analyzed. For this purpose a number of methods have been proposed, such as based on filtering converters, on a definite number of converters, or by fastening the bodies with known geometry of profile to the bodies the vibrodisplacements of which are being measured.

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**Non-contact control of velocity.** The essence of topological method of measurement of the stepwise linear velocity is the following.

Suppose the triangle $ABC$ is moved stepwise in the direction of the axis $Ox$ with the velocity $v = \dot{x}(i)$ (Fig. 12). The data of the triangle $ABC$ are the following: $AB = AC$, $\angle BAC = \alpha$, $BC$ is parallel to the axis $Ox$.

At equal time intervals $\Delta t$ in Fig. 12 we denote the positions of the triangle $ABC$ by dashed lines. All the sequential positions of the triangle we denote sequentially by numbers 1, 2, ..., $i-1$, $i$, $i+1$, ... and the vertices of the triangle in various positions by the letters $A_i$, $B_i$ and $C_i$ with the corresponding subscripts ($i = 1, 2, 3, ...$ – an integer positive number).

The intersection of the two opposite sides in the neighboring positions of the triangle is denoted by the letters $D_{i,i+1}$ ($i = 1, 2, ...$). The distance between the points $A_i$ and $A_{i+1}$ is denoted as $\Delta x_{i,i+1} = A_iA_{i+1}$.
The line segment $\overline{D_{i+1}E_{i+1}}$ perpendicular to the axis $Ox$ is denoted as $H_{i+1} = D_{i+1}E_{i+1}$.

The following linear relationship exists between $\Delta S_{i+1}$ and $H_{i+1}$:

$$\Delta S_{i+1} = 2H_{i+1} \tan \frac{\alpha}{2}. \quad (77)$$

Because the positions 1, 2, ..., $i-1$, $i$, $i+1$, ... of the triangle $ABC$ were marked in the process of its motion along the $Ox$ axis at equal time intervals $\Delta t$, the relationship:

$$v_{i+1} = \frac{\Delta S_{i+1}}{\Delta t}, (i = 1, 2, \ldots). \quad (78)$$

will give the average velocity of motion of the triangle $ABC$ from the position $i$ to the position $i+1$.

From eq. (77) and eq. (78) we find:

$$v_{i, i+1} = \frac{2 \tan \frac{\alpha}{2}}{\Delta t} H_{i, i+1}, (i = 1, 2, \ldots). \quad (79)$$

In Fig. 3 we connect all the points $D_{i, i+1}$ ($i = 1, 2, \ldots$) and obtain a graphical relationship $H_{i, i+1}$ or in the scale $\frac{2 \tan \frac{\alpha}{2}}{\Delta t}$ a graphical relationship of the averaged velocity of motion of the triangle $ABC$. The smaller the $\Delta t$ the nearer is the graphical relationship of the averaged velocity to the graphical relationship of the true velocity.

Further this principle is used for the measurement of the stepwise velocity of the links of the mechanisms in the following way.

To the link of the mechanism the stepwise velocity of motion of which is being measured we attach a white screen with a black triangle on it; it is possible to do it in an opposite way, namely: a black screen with a white triangle.

During the time of measurement of the velocity of motion of the analyzed link of the mechanism the moving screen with the triangle is lightened impulsively at equal time intervals (for example by a stroboscope). If the impulsively lightened moving screen with the triangle is photographed with large exposition, then on the photo we obtain darker or lighter triangles $A_iD_{i+1}$ (in the same way as in Fig. 12). By connecting the vertices $D_{i, i+1}$ of those triangles we obtain the graphical relationship (in the form of ordinates in the upper direction from the axis $Ox$) of the velocity of motion of the analyzed link of the mechanism as a function of the displacement.

This method is suitable for the measurement of velocities in the periodic and transitional regimes.
In case when the velocity of the link of the mechanism changes periodically it is possible to perform the analysis without the photographic devices. In those cases at sufficient frequency of periodicity of motion of the link of the mechanism because of the inertia of human eyes the graphical relationship of the velocity is well observed visually. In separate positions the velocity of the link of the mechanism can be measured directly on the lightened screen.

It is evident that various modifications of this method and its generalization for more complicated cases are possible. The angular velocity is also measured in the same way. Only in this case instead of the triangle two opposite Archimedes spirals going out from the center of the axis of rotation are used.

This method of measurement of velocity was used by us when conducting a number of experiments with mechanisms located on a vibrating basis. For example in Fig. 13 the graphical relationship of the velocity of motion of the pusher of the sine mechanism the rotor of which because of the harmonic vibration of the foundation of the mechanism is rotating non-uniformly is shown.

Fig. 13. Approximate graphical relationships of the stepwise velocity obtained by the topological method

**Methods and means of experimental investigation of dynamics of precise tape drive mechanisms (coauthor).** The proposed methods and measurement devices are developed for the most complicated case, when the investigated body is flexible with uneven surface and in the process of motion performs spatial vibrations. As an example of such a problem serves a moving uneven tape in precise tape drive mechanisms (TDM), which are applied for example in the devices for precise magnetic recording.

General case of behavior of flexible uneven moving bodies in space and their interaction with elements of mechanisms have been analyzed. Spatial vibrations of such bodies have been investigated and the problem of their vibro-measurement was formulated. It was determined that the most general case in the measurements may be considered to be the measurements of transverse vibrations of a moving uneven body. The problem becomes especially complicated when non-contact measurements are performed, because in this case there is no basis from which to measure the data.

It was determined how the general case of vibro-measurements of transverse vibrations of a moving tape with uneven borders in the TDM may be applied for the determination of vibrations of other types and also of the relationships between different types of vibrations of the tape, namely: between the tension and torsion, between the tension, transverse and perpendicular vibrations. Results of experimental investigations of those relationships have been obtained.

A new class of devices for the creation of directional vibrations of magnetic tape with respect to the blocks of recording and playing heads was developed. The necessity of such devices is based on the fact that the determination of correlation dependence on the basis of the existing mathematical models between a number of inputs, each of which represents a separate
type of mechanical vibrations, and the output, which determines one or several types of precision parameters of TDM, is a complicated process of investigation with insufficiently precise results. By using the developed devices it is possible to perform experimental and semi-physical investigations of the quality of precise TDM. Among the various directional vibrations of the tape longitudinal, transverse, perpendicular, rotational vibrations, turning and pulsation of the envelope angle were investigated. Schematic representations of devices, which enable to create a wide frequency band: from fractions of Hertz up to ultrasound frequencies, were proposed.

It was determined how various types of devices for measurement of vibrations of a moving tape with uneven borders may be applied for the investigation of different objects.

The possibility of application of the methods and devices of measurement of mechanical vibrations of TDM for protection from disturbances, for example from false signals in the process of magnetic recording on the tape, drum and disk, which arise because of spatial vibrations in the magnetic recording devices, was investigated.

Investigations of a separate type of vibrations – pulsation of tension of the tape were performed. Methods and devices for measurement of tension of flexible bodies by taking into account specific problems related with precise TDM were developed, namely: requirement of small forces in the tape, required wide frequency band of measured tensions, possibility of avoidance of distortion of paths of motion of flexible bodies. This type of vibrations deserves great attention, because the pulsation of tension is a source of excitation of other types of vibrations. Contact and non-contact methods of measurement of constant and variable components of tension of flexible tapes have been investigated, for example magnetic, and various devices were created and also calculation of errors of measurement was performed. The proposed methods and devices were applied for the investigation of dynamics of TDM. Dynamic dampers of vibrations of tension of a moving tape were created.

Other applications of vibration engineering (coauthor)

They are mostly investigations of a hobby type.

They are mostly investigations related with vibrations and waves in the fields of biology, medicine, agriculture.

The device for measurement of variation of air flow rate which is being inhaled into and exhaled from the human lungs has been produced.

The device for excitation of vibrations of the inhaled air has been produced in order to excite resonance vibrations of the lungs (bronchs, alveols).

The method for excitation of directional wave processes in lungs has been created.

The method for excitation of travelling waves in blood arteries of legs has been created in order to intensify the return of blood to the heart.

The method for abolishment of harmful stones from kidney channels by using vibrations has been created.

The vibro-platform for abolishment of tiredness of a human being has been produced.

Vibrational devices for the extermination of microbes inside the teeth have been produced.

A microrobot for performing cuts on the surface of the eyes has been produced.

The change of transmission of optical and sound signals by the eyes and the ears to the human nervous system into the action of external vibrations to the organism through the skin was investigated and some devices were produced.

The device for the measurement of vibrations of insects which are located inside the grains has been produced. Thus the type of insects and the percent part of the contaminated grains are determined.

Some effects of dynamical synchronization in living organisms have been developed.
Conclusions

1. The methods of investigation of some types of essentially nonlinear mechanical systems have been developed and some dynamical effects and qualities of those systems have been revealed (dynamical multiple synchronization, dynamical self-organization of structures, dynamical directivity, methods of transformation of motions and energies).

2. Methods for the construction of mechanical systems operating on the basis of new principles based on the interaction of vibrations and waves and nonlinear elements have been created.

3. Methods for stabilization and deestructurization of structures of some nonlinear systems have been created.

4. Methods of measurement and control of vibrations of precise mechanical systems separating vibrations from disturbances which arise due to non-uniformities of the surfaces of bodies, due to deformations, due to external excitations have been created.

5. On the basis of the results of investigations doctoral dissertations have been prepared, inventions and innovations have been developed.

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