596. Amplitude of Radial Oscillations and Imbalance Parameters of the Rotating Rotor

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Abstract. The paper proposes a method of continuous definition of imbalance magnitude of a rotating rotor in process of its operation on the basis of analysis of amplitude of the radial fluctuations, which is measured (identified) as a distance between rotation and symmetry axes of a rotor.

Keywords: rotor imbalance, amplitude of radial oscillations, radial displacement, axis of rotation, symmetry axes.

The amplitude of radial oscillations of a rotating rotor may be determined in the context of continuous systemic definition of parameters [1-3] as distance between axes of rotation and symmetry of a rotor (Fig. 1) in the form [4-6]:

\[ A_0 = \sqrt{R^2 + r_3^2 - 2R r_3 \cos(\alpha - \beta)}, \]  

(1)

where radius of a rotor

\[ R = \frac{1}{2} \sqrt{\frac{r_2^2 + r_3^2 - 2r_2 r_3 \cos \chi_{23}}{1 - \frac{r_1^2 - r_1 r_2 \cos \chi_{12} - r_1 r_3 \cos \chi_{13} + r_2 r_3 \cos \chi_{23}}}}; \]  

(2)

\[ \alpha = \arcsin \frac{r_1^2 - r_1 r_2 \cos \chi_{12} - r_1 r_3 \cos \chi_{13} + r_2 r_3 \cos \chi_{23}}{\sqrt{(r_1^2 + r_2^2 - 2r_1 r_2 \cos \chi_{12})(r_1^2 + r_3^2 - 2r_1 r_3 \cos \chi_{13})}}; \]  

(3)

\[ \beta = \arcsin \frac{r_2 \sin \chi_{23}}{\sqrt{r_2^2 + r_3^2 - 2r_2 r_3 \cos \chi_{23}}}; \]  

(4)

\( r_1, r_2, r_3 \) – radiuses-vectors of points A, B, C on a surface of the rotor, which are controlled by sensors \( D_1, D_2, D_3 \) accordingly (Fig. 1);

\( \chi_{12}, \chi_{23}, \chi_{31} \) – angles between radiuses-vectors \( r_1, r_2, r_3 \);

\( D_1, D_2, D_3 \) – sensors which supervise distances \( s_1, s_2, s_3 \) up to opposite points A, B, C on surfaces of a rotor and also linear speeds \( V_1, V_2, V_3 \) of these points.
Geometrical construction, presented in Fig. 1, b, allows to compose and solve the equations system in which radiuses-vectors $r_1$, $r_2$, $r_3$ and angles $\chi_1$, $\chi_2$, $\chi_3$ between them are decision variable, and directly measured by sensors $D_1$, $D_2$, $D_3$ magnitudes $s_1$, $s_2$, $s_3$, $V_1$, $V_2$, $V_3$ are coefficients changing at a rotor rotation [2, 6].

![Fig. 1. Distance definition between axes of rotation and symmetry of a rotor](image)

It is considered that due to rotor imbalance its rotation occurs around the axis which passes through the center of masses, and the amplitude of radial oscillations $A_0$ is equal to mass eccentricity $\varepsilon$ (i.e., $A_0 = \varepsilon$). We will further demonstrate that such statement is fair only for angular rotor speed $\omega$, which is significantly smaller than resonant value $\Omega$ (at condition $\omega \ll \Omega$).

In the sufficiently large values of angular speed $\omega$ displacement of the mass center of the rotor concerning the symmetry axis, which is caused by its imbalance, leads to additional inertial displacement $\lambda$ of the rotation axis concerning the mass center [7-10]. In this case the amplitude of radial fluctuations can be represented in the following form:

$$A_0 = \varepsilon + \lambda.$$

(5)

Magnitude of inertial displacement $\lambda$ for harmonic oscillations can be determined as [7, 8, 10]:

$$\lambda = \frac{\varepsilon \omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2 \omega^2}},$$

(6)

where $\mu$ is the damping coefficient of oscillations. We may find from (5) an allowance expression (6):

$$A_0 = \varepsilon \left[ 1 + \frac{\omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2 \omega^2}} \right].$$

(7)

The analysis of expression (7) indicates that under condition of $\Omega > \omega$ magnitude

$$\lambda = \frac{\varepsilon \omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2 \omega^2}} > 0,$$

and, consequently

$$A_0 = (\varepsilon + \lambda) > \varepsilon.$$

(8)
It follows from (8), in the area of under-resonant frequencies amplitude $A_0$ of radial oscillations exceeds mass eccentricity of a rotor.

Under condition of $\Omega > \omega$ magnitude

$$\lambda = \frac{\varepsilon \omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2\omega^2}} < 0,$$

and, consequently

$$A_0 = (\varepsilon + \lambda) < \varepsilon.$$  \hspace{1cm} (9)

It follows from (9), in the area of after-resonant frequencies amplitude $A_0$ of radial oscillations becomes smaller mass eccentricity of a rotor.

Let’s define limiting values of inertial displacement $\lambda$ and amplitudes $A_0$ of radial oscillations.

$$\lim_{\omega \to 0} \lambda = \lim_{\omega \to 0} -\frac{\varepsilon \omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2\omega^2}} = 0; \hspace{1cm} (10)$$

$$\lim_{\omega \to 0} A_0 = \lim_{\omega \to 0} \varepsilon \left[ 1 + \frac{\omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2\omega^2}} \right] = \varepsilon. \hspace{1cm} (11)$$

It follows from (10) and (11), under condition $\omega \to 0$ and, accordingly, $\lambda_{\omega=0} \to 0$, amplitude $A_0$ of radial oscillations approaches to magnitude of mass eccentricity $\varepsilon$ of a rotor:

$$A_0\big|_{\omega \to 0} \to \varepsilon. \hspace{1cm} (12)$$

Magnitude $\text{sign}(\Omega^2 - \omega^2) < 0$ at infinitely large increase of angular speed $\omega$ (and, hence, under condition $\Omega < \omega$), and we may find from (6) and (7):

$$\lim_{\omega \to \infty} \lambda = \lim_{\omega \to \infty} -\frac{\varepsilon \omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2\omega^2}} = -\varepsilon; \hspace{1cm} (13)$$

$$\lim_{\omega \to \infty} A_0 = \lim_{\omega \to \infty} \varepsilon \left[ 1 + \frac{\omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - \omega^2)^2 + 4\mu^2\omega^2}} \right] = 0. \hspace{1cm} (14)$$

Dependence of inertial displacement $\lambda$ (a) and amplitude $A_0$ of radial oscillations (b) from angular speed $\omega$ and mass eccentricity $\varepsilon$ is presented in Fig. 2.

![Fig. 2. Dependence of inertial displacement $\lambda$ and amplitude $A_0$ of radial oscillations from angular speed $\omega$ and mass eccentricity $\varepsilon$.](image-url)
It follows from (13) and (14), under condition $\omega \to \infty$ and, accordingly, $\lambda_{\omega \to \infty} \to -\varepsilon$, amplitude $A_0$ of radial oscillations has approximation to zero value. It testifies to approximation of rotation axis of a rotor to its symmetry axis (so-called effect of a self-centering of a high-speed rotor).

The trajectory of point $G_0$ of crossing of rotation axis of the high-speed rotor ($\omega_0 > 1000 \text{ s}^{-1}$) with a supporting plane $0xy$ (at that the point $0$ is a point of crossing of the symmetry axis of a rotor with plane $0xy$) is provided in Fig. 3 [11, 12]. Conditions $t \to 0$ and $\omega \to 0$ occur at the starting moment. According to (10) and (11), the distance $0G_0$ is distance between rotation axis and symmetry axis (amplitude of radial oscillations $A_0$), is equal to mass eccentricity $\varepsilon$ of a rotor. The increase of angular speed in limits $\Omega > \omega > 0$ entails increase of amplitude of radial oscillations: $A_0 > \varepsilon$. After a resonance passage the amplitude of radial oscillations $A_0$ are reduced and at exceeding by angular speed of a resonant threshold (the condition $\omega > \Omega$) it is become less of mass eccentricity $\varepsilon$.

We may define mass eccentricity of a rotor by comparison of expressions (1) and (7):

$$\varepsilon = \frac{\sqrt{R^2 + r_3^2 - 2Rr_1 \cos(\alpha - \beta)} - \omega \varepsilon \frac{\omega_0 \varepsilon \lambda_{\omega_0 \to \infty}}{\Omega - \omega^2}}{1 + \frac{\omega \varepsilon \lambda_{\omega_0 \to \infty}}{\sqrt{(\Omega - \omega^2)^2 + 4\mu^2 \varepsilon^2}}},$$

where magnitudes $R$, $\alpha$, and $\beta$ are defined by expressions (2), (3) and (4).

Momentary value of angular speed $\omega$ of a rotor can be determined in the form:

$$\omega = \frac{V_1}{r_1} = \frac{V_2}{r_2} = \frac{V_3}{r_3}$$

where $r_1$, $r_2$, $r_3$ are momentary values of radiuses-vectors of supervised points A, B, C on a rotor surface and $V_1$, $V_2$, $V_3$ are linear speeds of these points which are measured directly by sensors $D_1$, $D_2$, $D_3$ [6, 10].

The damping coefficient $\mu$ of oscillations may be defined in the form [13]:

$$\mu = \delta \frac{\Omega^2 - \omega^2}{2\omega} \sqrt{\frac{1}{\pi^2 - \delta^2}},$$

where logarithmic decrement of oscillation attenuation
\[ \delta = \ln \frac{A_{1\text{res}}}{A_{2\text{res}}} , \]  
(18)

\( A_{1\text{res}} \) and \( A_{2\text{res}} \) are the amplitudes of two consecutive oscillations which are divided by period \( T \) and measured at a resonance passage (oscillations, which are approximate to the free radial oscillations of a rotor).

Dependence of the damping coefficient \( \mu \) on angular speed \( \omega \) and logarithmic decrement \( \delta \) attenuations of fluctuations is given in Fig. 4.

**Fig. 4.** Variation of damping coefficient \( \mu \) as a function of angular speed \( \omega \) and logarithmic decrement of attenuation \( \delta \)

We find from expression (15) by taking into account (16) and (17):

\[ \varepsilon = \frac{\sqrt{R^2 + r^2 - 2Rr \cos(\alpha - \beta)}}{1 + \frac{V_2^2}{\pi \Omega^2 \Omega^2 - V_2^2}} . \]  
(19)

The analysis of expression (19) indicates that mass eccentricity of a rotating rotor can be measured continuously, in process of its operation, at continuous measurement of linear speeds \( V_1, V_2, V_3 \) of supervised points A, B, C on a rotor surface and identification of radiuses-vectors \( r_1, r_2, r_3 \) of these points\(^1\) [6].

If the rotor oscillations are periodic nonharmonic oscillations, the value of inertial displacement \( \lambda \) with consideration of Fourier series may be determined in the form [10]:

\[ \lambda = \sum_{k=1}^{\infty} \frac{\varepsilon k^2 \omega^2 \text{sign}(\Omega^2 - \omega^2)}{\sqrt{(\Omega^2 - k^2 \omega^2)^2 + 4 \mu^2 k^2 \omega^2}} , \]  
(20)

where \( k \) is the harmonic order of Fourier series.

We may find from equation (5) with regard to expression (20):

\(^1\) The choice of magnitudes \( r_3 \) and \( V_3 \) in formula (19) is determined by a casual choice of geometrical construction (Fig. 1, b). Other identical geometrical constructions define a choice of magnitudes \( r_1 \) and \( V_1 \), or magnitudes \( r_2 \) and \( V_2 \).
In the case of periodic nonharmonic oscillations mass eccentricity $\varepsilon$ of a rotor may be determined from comparison of expressions (1) and (21):

$$
\varepsilon = \sqrt{R^2 + r_i^2 - 2Rr_i \cos(\alpha - \beta)}
$$

$$
+ \sum_{k=1}^{\infty} \frac{k^2 V_i^2}{\pi \Omega^2 r_i^2 - k^2 V_i^2}
$$

where $i$ is the number of one of three radiuses-vectors of points A, B, C on a rotor surface and $V_i$ is linear speed of corresponding point.

It follows from the presented study that continuous measurement (identification) of mass eccentricity is provided with system of integrated measurement of mechanical parameters of a rotor [1-3] and does not require application of the special measuring elements and devices.

References


