595. The Analytical Method of Acoustic Field Estimation in the Cylindrical Shape

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Abstract. This paper studies directional characteristics of acoustic field in cylindrical shape. The main object of this paper is to determine the impulse response of sound sources for calculating acoustics parameters in cylindrical closed space. The obtained results demonstrate that application of the proposed analytical model enables analysis of the formed sound field of the sound sources acting at the certain frequency by modeling the acoustic field in cylindrical shape.

Keywords: acoustic field, analytical method, estimation

Firstly, a model of a cylindrical shape is constructed as indicated in Fig. 1. Within is air particles, shown as circles and a distance between particles is equal to $\lambda/2$. So the motion of particles with respect to the frame is relative. The number and location of particles depend on known frequency ν_k and speed of sound in air c, because λ_k – known wavelength:

$$\lambda_k = \frac{c}{\nu_k} \tag{1}$$

Let's suppose that:

- walls of cylinder is absolutely rigid and in equilibrium;
- the sound propagation is adiabatic;
- a sound source characteristics and geometric measurements are known;
- the external volume (air mass) forces are ignored.

We shall choose a system of axes for the mathematical model and displacements (see Fig. 2).

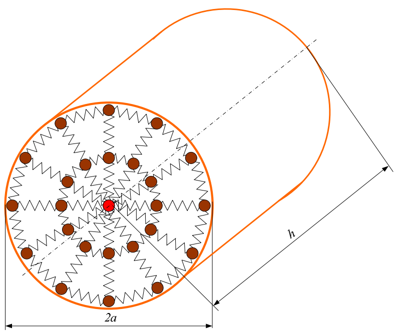

Fig. 1. Model of air in cylindrical shape

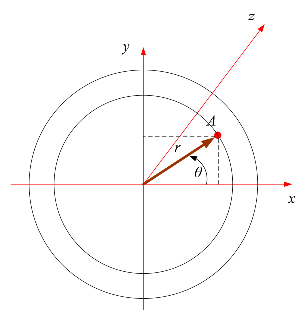

Fig. 2. Displacement counting diagram

Let's suppose that displacements towards *Ox* direction are $u = r\cos\theta$, towards *Oy* direction – $v = r\sin\theta$, towards *Oz* direction – w in cylindrical system of axis. We are going to find displacements *u*, *v* and *w* for the boundary conditions:

- displacements at the wall are equal to zero, thus when $r = \pm a$, then displacements u=0, v=0 and z=0; z=h, then w=0.
- on the wall surfaces, the partial derivative of relative displacement by normal are equal to zero;

and in case of initial conditions:

- u=v=w=0 displacements of the air points in the initial moment t=0 are equal to zero.

Then kinetic energy of air particles is expressed:

$$T = \frac{\rho_o}{2} \iiint\limits_{(V)} \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) r dr d\theta dz \ , \tag{2}$$

where

ρ_o – density of air;

S – integration area, i. e. the space part occupied by air.

Potential energy of external surface forces (sound source)

$$U_p = \iint\limits_{(S)} \left(\bar{X}u + \bar{Y}v + \bar{Z}w \right) dS \ , \tag{3}$$

where

$\bar{X}, \bar{Y}, \bar{Z}$ – projections of external surface force (line unit is subjected to that force) on co-ordinate axes;

S – integration area, i.e. the surface part subjected to external surface forces.

Potential energy of air deformation forces:

$$U_T = -\frac{\rho_o}{2c^2} \iiint\limits_{(v)} \omega r dr d\theta dz \tag{4}$$

$$\omega = \left(\frac{\partial u}{\partial r} \right)^2 + \left(\frac{\partial v}{r \partial \theta} + \frac{u}{r} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \tag{5}$$

In that case Hamilton principle is expressed by equation:

$$\delta \int_{t_1}^{t_2} \left(T + U_P + U_T \right) dt = 0 \, . \tag{6}$$

Substituting expression (2-5) for (6), we get

$$\delta \int_{t_1}^{t_2} \left\{ \begin{aligned} & -\frac{\rho_o}{2c^2} \iiint\limits_{(v)} \left[\left(\frac{\partial u}{\partial r} \right)^2 + \left(\frac{\partial v}{r \partial \theta} + \frac{u}{r} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] r dr d\theta dz + \\ & + \frac{\rho}{2} \iiint\limits_{(V)} \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) r dr d\theta dz + \iint\limits_{(S)} \left(\bar{X}u + \bar{Y}v + \bar{Z}w \right) dS \end{aligned} \right\} dt \ . \tag{7}$$

Let's suppose that separation function:

$$u = Uq\ ;\quad v = Vq\ ;\ w=Wq, \tag{8}$$

where

$$U = U\left(r,\theta,z\right);\quad V = V\left(r,\theta,z\right);\quad W = W\left(r,\theta,z\right);\quad q = q\left(t\right).$$

The mathematical model is obtained from Hamilton principle from equation (7) will transform into and giving the following equation:

$$-\rho_o \ddot{q}\iiint\limits_{(V)}\left(U^2+V^2+W^2\right)rdrd\theta dz - \frac{\rho_o}{c^2}\iiint\limits_{(V)}\left[U_r^2+\left(V_\theta+\frac{U}{r}\right)^2+W_z^{\,2}\right]rdrd\theta dz + \\ +\iint\limits_{(S)}\left(\bar{X}U+\bar{Y}V+\bar{Z}W\right)dS = 0 \tag{9}$$

where

$$\frac{\partial U}{\partial r} = U_r\,,\ \frac{\partial V}{r\partial\theta} = V_\theta\,,\ \frac{\partial W}{\partial z} = W_z\,,\ \frac{\partial U}{\partial z} = U_z\,.$$

This equation can be rewritten:

$$m_o\frac{dq}{dt} = -k_o\int_0^\tau q dt + \int_0^\tau P dt\,, \tag{10}$$

where

$$m_o = \rho_o\ddot{q}\iiint\limits_{(V)}\left(U^2+V^2+W^2\right)rdrd\theta dz\,, \tag{11}$$

$$k_o = \frac{\rho_o}{c^2}\iiint\limits_{(V)}\left[U_r^2+\left(V_\theta+\frac{U}{r}\right)^2+W_z^{\,2}\right]rdrd\theta dz \tag{12}$$

$$P = \iint\limits_{(S)}\left(\bar{X}U+\bar{Y}V+\bar{Z}W\right)dS\,. \tag{13}$$

$$\tau_i = \frac{1}{4\nu_i}\,. \tag{14}$$

Equation (10) can be solved approximately by means of iteration method, for instance [1]:

$$q = \frac{LI}{m_o}t - \frac{k_o LI}{6m_o^2}t^3 + \frac{k_o^2 LI}{120m_o^3}t^5 - \frac{k_o^3 LI}{5040m_o^4}t^7 + \frac{k_o^4}{362880m_o^4}t^9 \tag{15}$$

where, in the case of sound source, taking into account that the pressure of sound is the same in all directions $(\bar{X} = \bar{Y} = \bar{Z})$,

$$I = \int_0^{\tau} \overline{X} dt = \int_0^{\tau} \overline{Y} dt = \int_0^{\tau} \overline{Z} dt \,.$$

$$\int_0^{\tau} P dt = IL = I \iint_{(S)} (U + V + W) dS \qquad (16)$$

Knowing m_o, k_o, L and taking into account that sound intensity (pressure) is inversely proportional to the square distance from the point of sound source, we can calculate the approximate value q according to the equations (15). Finally, having applied equation (8), we can calculate approximate relative displacements of air particles.

The problem simulated numerically is provided in Fig. 1. For example, the geometrical values a=3 m and h= 4 m. Let's suppose that density of air $\rho_o = 1.224$ kg/m^3, speed of sound in air is $c = 343$ m/s and functions:

Figs. 3-4 illustrate fragments of calculation results of acoustics field of frequency $\nu = 8000$ Hz when the sound source located at different place shape.

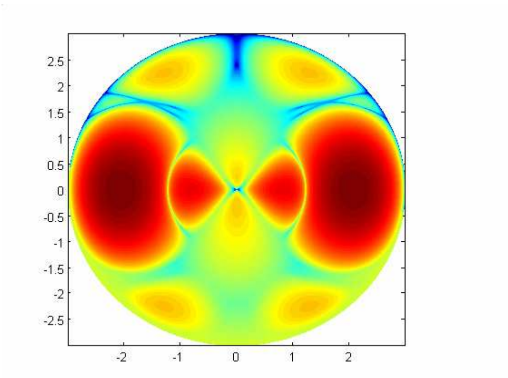

Fig. 3. Acoustics field when the sound source is located at the center of shape

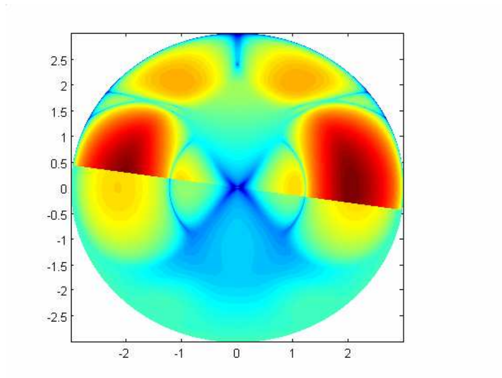

Fig. 4. Acoustics field when the sound source is located at $r = 2$ m

Conclusions

The proposed analytical method allows analysis of interaction of dynamic processes of a sound source with air in the cylindrical shape and enables to create precondition for estimating acoustics field in an enclosure.

References

[1] **Doroševas, V.** The new analytical method of acoustic field estimation in the room. // Journal of Vibroengineering / Vibromechanika. Vilnius: Vibromechanika. ISSN 1392-8716. 2008, Vol. 10, No.3, p. 302-306.