504. Motion control optimization of robotic fish tail

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Abstract. In the daily life people, animals, fishes, birds and insects constantly interact with continuous media such as air or water. It means that synthesis of new robotic systems inside this continuous media and imitation of motion of real objects must be investigated together with media surrounding them. In this report motion of robotic fish tail vibration and simplified interaction with water flow is investigated. The study comprises: (a) preliminary analysis of main goal that is given to scientist to solve the problem; (b) optimization of a main fundamental system; (c) analysis of ideal control actions; (d) synthesis a new structural schemes; (e) calculation of optimal parameters. The preliminary analysis includes 5 steps: analysis of technological processes, choice of base system, choice of control actions, clarification of criteria for optimization and selection of limits. In this report criterion of optimization (for robotic fish tail model inside water) is maximal positive impulse of water forces in the fish tail and hull contact pivot. The main idea is to find out optimal control law for variation of additional area of vibrating tail within limits. The limits are maximal and minimal area of tail interaction with water. For task solution the maximum principle of Pontryagin is used [5 – 14]. It is demonstrated that optimal control action is on bounds of area limits. Examples of synthesis of real mechatronic systems are provided. One example of synthesis is a system with time harmonic moment excitation of tail in the pivot. The other example is a system with adaptive force moment as function of phase coordinates. In both systems area exchange (from maximal to minimal values) has control action as a function of phase coordinates. It is demonstrated that real tail vibration motion is highly stable and provides satisfactory real criterion in the hull contact point.

Keywords: Robotics fish, motion control, water interaction, optimal control, adaptive control, synthesis adaptive systems.

Introduction

Inverse method algorithm for development of mechatronic systems in vibrotechnics is used for robotic fish tail motion control synthesis. The main difference of this method in comparison to simple analysis method is that (before synthesis of real system) optimal control task for abstract initial subsystem is solved [1 - 4]. After some calculation optimal control law is determined that allows synthesis of series of structural schemes for real systems right initial subsystem. It is shown that near optimal control excitation new structural schemes may be found in the medium of three kinds of strongly non-linear systems: - systems with excitation as a time function; - systems with excitation as a function of phase coordinates only; - systems with both mixed excitations [2 – 4]. In this report synthesis of systems with adaptive excitations are analyzed.

Equations of motion

Simple robotic fish model consist of hull and tail like rigid bodies (Fig. 1.). If the hull is large and moves slowly with stirring along x axis but the tail is vibrating very quickly then a simplified model with one degree of freedom - tail angle φ - may be analyzed (Fig. 2.).

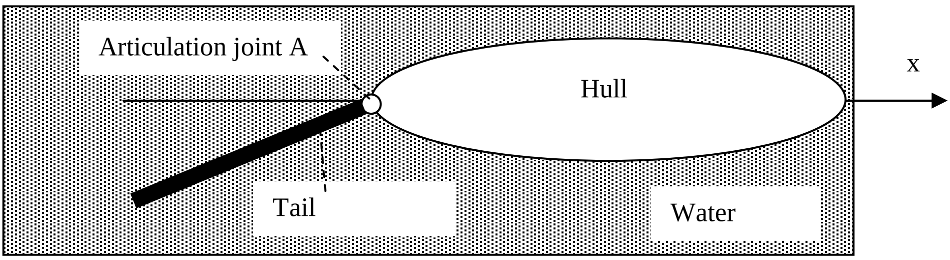


Fig. 1. Scheme of full model: hull, tail and joint inside water

Mathematical model of system consists of rigid straight flat tail moving around pivot (Fig. 2.). Area of tail may be changed by control actions. Moment *M(t,φ,ω)* around pivot must be added for motion excitation, where $\omega = d\varphi/dt$ – angular velocity of rigid tail (Fig. 3.). For more resources of system additional rotational spring with stiffness *c* may be added (Fig. 4.).
The main idea in this report is to find out optimal control law for variation of additional area *B(t)* of vibrating tail within limits (1.):

$$B_1 \le B(t) \le B_2, \qquad (1)$$

where B_1 - lower level of additional area of tail; B_2 - upper level of additional area of tail, *t* - time.

The criterion of optimization is full reaction *Ax** impulse in pivot (2), acted to hull (indirect reaction to tail):

$$K = -\int_0^T Ax^* \cdot dt\,, \qquad (2)$$

required to move object from one stop initial position ($\varphi_{max}, \dot{\varphi} = 0$) to second stop end position ($\varphi_{min}, \dot{\varphi} = 0$) in time *T*.

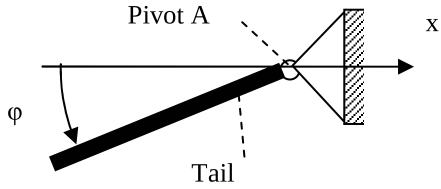


Fig. 2. Simplified model with fixed pivot

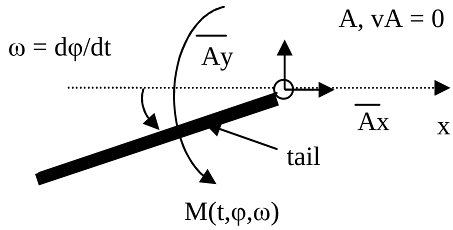


Fig. 3. Scheme of moment $M(t,\varphi,\omega)$ and pivot reactions $Ax = - Ax^*, Ay$

Scheme of nonlinear tail interaction with water (includes resistance forces) is presented in Fig. 5.

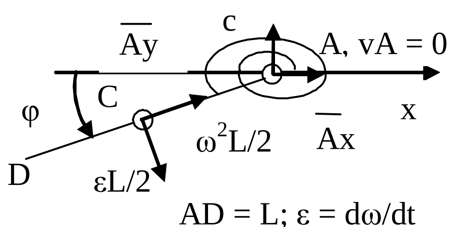


Fig. 4. Scheme of pivot spring and acceleration of centre mass for rectilinear tail

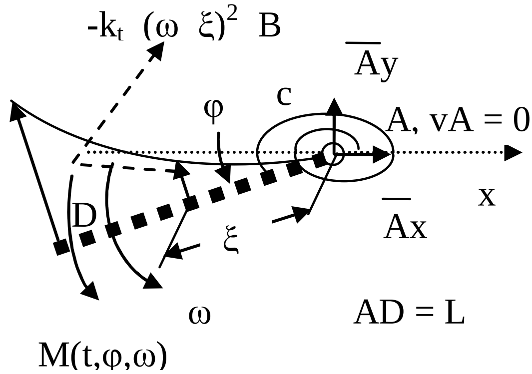


Fig. 5. Scheme of nonlinear tail interaction with water (velocity in square): ξ – distance from pivot; k_t – constant; B – area

The differential equation of motion of system with one degree of freedom (by use exchange of moment of momentum around fixed point A) is:

$$J_A\ddot{\varphi} = M(t,\varphi,\dot{\varphi}) - c\cdot\varphi - k_t\cdot B\cdot sign(\dot{\varphi}\cdot\varphi)\cdot\int_0^L(\dot{\varphi}\cdot\xi)^2\cdot\xi\cdot d\xi. \quad (3)$$

Here

$J_A, \ddot{\varphi}, \int_0^L(\dot{\varphi}\cdot\xi)^2\cdot\xi\cdot d\xi$, L - moment inertia tail mass against pivot point; angular acceleration of rigid body straight tail; component of moment of resistance force like integral along tail direction; length of tail.

From theorem of tail mass *m* center *C* motion follows (4) (Fig. 4.):

$$m\cdot\left(\dot{\varphi}^2\cdot\frac{L}{2}\cdot\cos\varphi+\ddot{\varphi}\cdot\frac{L}{2}\cdot\sin\varphi\right)=Ax-k_t\cdot B\cdot\sin(\varphi)\cdot sign(\varphi\cdot\dot{\varphi})\cdot\left(\int_0^L(\dot{\varphi}\cdot\xi)^2\cdot d\xi\right) \quad (4)$$

After integration, from equations (3) and (4) we have (5, 6):

$$\ddot{\varphi}=\frac{1}{J_A}\cdot\left[M(t,\varphi,\dot{\varphi})-c\cdot\varphi-k_t\cdot B\cdot sign(\dot{\varphi})\cdot\dot{\varphi}^2\cdot\frac{L^4}{4}\right]. \quad (5)$$

$$Ax=m\cdot\{\dot{\varphi}^2\cdot\frac{L}{2}\cdot\cos(\varphi)+\frac{1}{J_A}\cdot\left[M(t,\varphi,\dot{\varphi})-c\cdot\varphi-k_t\cdot B\cdot sign(\dot{\varphi})\cdot\dot{\varphi}^2\cdot\frac{L^4}{4}\right]\cdot\frac{L}{2}\cdot\sin(\varphi)\}+$$
$$+k_t\cdot B\cdot\sin(\varphi)\cdot sign(\varphi\cdot\dot{\varphi})\cdot\dot{\varphi}^2\cdot\frac{L^3}{3}. \quad (6)$$

Then the criterion of optimization is:

$$K=-\int_0^T\left(\begin{array}{l} m\cdot\{\dot{\varphi}^2\cdot\frac{L}{2}\cdot\cos(\varphi)+\frac{1}{J_A}\cdot\left[M(t,\varphi,\dot{\varphi})-c\cdot\varphi-k_t\cdot B\cdot sign(\dot{\varphi})\cdot\dot{\varphi}^2\cdot\frac{L^4}{4}\right]\cdot\frac{L}{2}\cdot\sin(\varphi)\}+\\ +k_t\cdot B\cdot\sin(\varphi)\cdot sign(\varphi\cdot\dot{\varphi})\cdot\dot{\varphi}^2\cdot\frac{L^3}{3}\end{array}\right)\cdot dt, \quad (7)$$

Last equations (7) will be used in task of optimizations.

Task of optimization

Solution of optimal control problem for system with one degree of freedom (5) by using the maximum principle of Pontryagin includes next steps [9 - 12]:

1. Formulation of criterion of optimization (6, 7):

$$K=-\int_0^T Ax\cdot dt\cdot$$

2. Transform the second order differential equation (5) and equation (7) in the three first order equations with new variables φ0, φ1 and φ2 (phase coordinates):

$$\dot{\varphi}0=(-1)\cdot\left[\begin{array}{l} m\cdot\{\varphi2^2\cdot\frac{L}{2}\cdot\cos(\varphi1)+\frac{1}{J_A}\cdot\left[M(t,\varphi1,\varphi2)-c\cdot\varphi1-k_t\cdot B\cdot sign(\varphi2)\cdot\varphi2^2\cdot\frac{L^4}{4}\right]\cdot\\ \frac{L}{2}\cdot\sin(\varphi1)\}+k_t\cdot B\cdot\sin(\varphi1)\cdot sign(\varphi1\cdot\varphi2)\cdot\varphi2^2\cdot\frac{L^3}{3};\end{array}\right]$$

$$\begin{aligned}&\dot{\varphi}1=\varphi 2;\\&\dot{\varphi}2=\frac{1}{J_A}\cdot\left[M(t,\varphi 1,\varphi 2)-c\cdot\varphi 1-k_t\cdot B\cdot sign(\varphi 2)\cdot\varphi 2^2\cdot\frac{L^4}{4}.\right]\end{aligned}\tag{8}$$

According to procedure of maximum principle of Pontryagin, Hamiltonian (H) is [9 -12]:

$$H=\psi_0\cdot(-1)\cdot\left[\begin{array}{l}m\cdot\}\varphi 2^2\cdot\frac{L}{2}\cdot\cos(\varphi 1)+\frac{1}{J_A}\cdot\left[M(t,\varphi 1,\varphi 2)-c\cdot\varphi 1-k_t\cdot B\cdot sign(\varphi 2)\cdot\varphi 2^2\cdot\frac{L^4}{4}\right]\cdot\\\frac{L}{2}\cdot\sin(\varphi 1)\})+k_t\cdot B\cdot\sin(\varphi 1)\cdot sign(\varphi 1\cdot\varphi 2)\cdot\varphi 2^2\cdot\frac{L^3}{3}\end{array}\right]+$$

$$+\psi_1\cdot\varphi 1+\psi 2\cdot\frac{1}{J_A}\cdot\left[M(t,\varphi 1,\varphi 2)-c\cdot\varphi 1-k_t\cdot B\cdot sign(\varphi 2)\cdot\varphi 2^2\cdot\frac{L^4}{4}\right].\tag{9}$$

here $H=\psi\cdot\Phi$, where (10, 11)

$$\psi=\begin{Bmatrix}\psi_0\\\psi_1\\\psi_2\end{Bmatrix};\tag{10}$$

$$\Phi=\begin{Bmatrix}\Phi_0\\\Phi_1\\\Phi_2\end{Bmatrix},\tag{11}$$

Φ_0,Φ_1,Φ_2 - right side of system equations (8).

For functions Φ_0,Φ_1,Φ_2 calculations are next differential equations [9 - 12]:

$$\dot{\Phi}_0=-\frac{\partial H}{\partial\varphi_0};\ \dot{\Phi}_1=-\frac{\partial H}{\partial\varphi_1};\ \dot{\Phi}_2=-\frac{\partial H}{\partial\varphi_2},\tag{12}$$

were left side derivation is from time t: $\dot{\Phi}_{0,1,2}=\dfrac{d\Phi_{0,1,2}}{dt}$.

From equations (9) and (12) we may determine non-linear system of differential equations to find functions ψ_0,ψ_1,ψ_2. This system solution is out of this report because depends of unknown moment $M(t,\varphi,\omega)$. But some conclusions and recommendations may be given from Hamiltonian ($M(t,\varphi,\omega)$) if excitation moment $M(t,\varphi,\omega)$ does not depend of phase coordinates $\varphi=\varphi 1,\ \omega=\varphi 2$:

$$M=M(t).$$

In this case scalar multiplication of two last vector functions ψ and Φ in any time (Hamiltonian *H* [3]) must be maximal (supremum - in this linear B case) [2 – 9]. To have such maximum (supremum), control action *B(t)* must be within limits $B(t) = B_1; \quad B(t) = B_2$, depending only from the *sign* of function *(12)or (13):*

$$sign\left(\begin{array}{l}\psi_0 \cdot \left[\dfrac{m}{J_A} \cdot \left[k_t \cdot 1 \cdot sign(\varphi 2) \cdot \dfrac{L^5}{4}\right] \cdot \dfrac{1}{2} \cdot \sin(\varphi 1) + k_t \cdot 1 \cdot \sin(\varphi 1) \cdot sign(\varphi 1 \cdot \varphi 2) \cdot \dfrac{L^3}{3}\right] + \\ + \psi 2 \cdot \dfrac{1}{J_A} \cdot \left[-k_t \cdot 1 \cdot sign(\varphi 2) \cdot \dfrac{L^4}{4}\right]\end{array}\right) \geq 0, \quad (12)$$

B = B2;

$$sign\left(\begin{array}{l}\psi_0 \cdot \left[\dfrac{m}{J_A} \cdot \left[k_t \cdot 1 \cdot sign(\varphi 2) \cdot \dfrac{L^5}{4}\right] \cdot \dfrac{1}{2} \cdot \sin(\varphi 1) + k_t \cdot 1 \cdot \sin(\varphi 1) \cdot sign(\varphi 1 \cdot \varphi 2) \cdot \dfrac{L^3}{3}\right] + \\ + \psi 2 \cdot \dfrac{1}{J_A} \cdot \left[-k_t \cdot 1 \cdot sign(\varphi 2) \cdot \dfrac{L^4}{4}\right]\end{array}\right) \leq 0. \quad (13)$$

B = B1;

From inequalities (12) and (13) in real system synthesis next quasi-optimal control action (see supremum of (9)) must be recommended and checked up:

$$B = \left[B2 \cdot (0{,}5 - 0{,}5 \cdot sign(\varphi 1 \cdot \varphi 2)) + B1 \cdot (0{,}5 + 0{,}5 \cdot sign(\varphi 1 \cdot \varphi 2))\right],$$

or

$$B = \left[B2 \cdot (0{,}5 - 0{,}5 \cdot sign(\varphi \cdot \dot{\varphi})) + B1 \cdot (0{,}5 + 0{,}5 \cdot sign(\varphi 1 \cdot \dot{\varphi})\right]. \quad (14)$$

Control action (14) analysis and synthesis is given in the following chapter.

Synthesis of mixed system with time harmonic excitation and area adaptive control

In a case of time harmonic excitation moment *M* in time domain is (see equations (3) and (9)):

$M(t) = M0 \cdot \sin(k \cdot t)$.

Results of modeling are illustrated in Fig. 6. – 11. Comments about graphics are given under all Fig. 6. – 11.

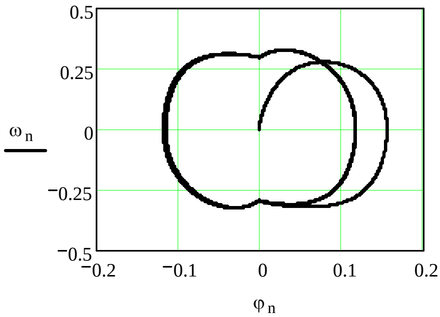


Fig. 6. Tail angular motion in phase plane – angular velocity as a function of angle. Because resistance of water is large transition process is very short

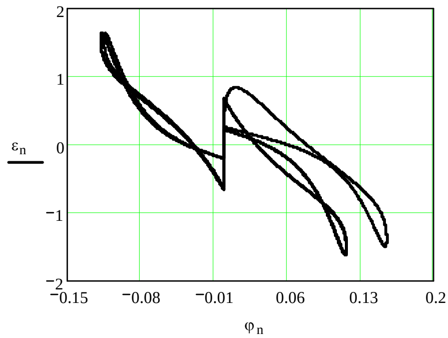


Fig. 7. Angular acceleration of tail as a function of angle. At the end of transition process graphic is symmetric

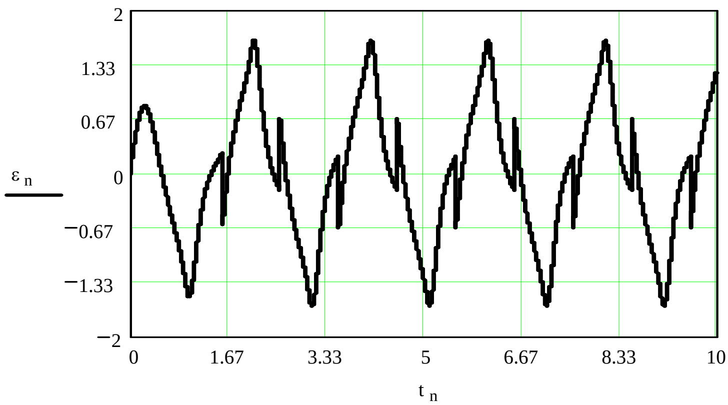


Fig. 8. Angular acceleration $\varepsilon = \ddot{\varphi}$ of tail in time domain (see equation (5))

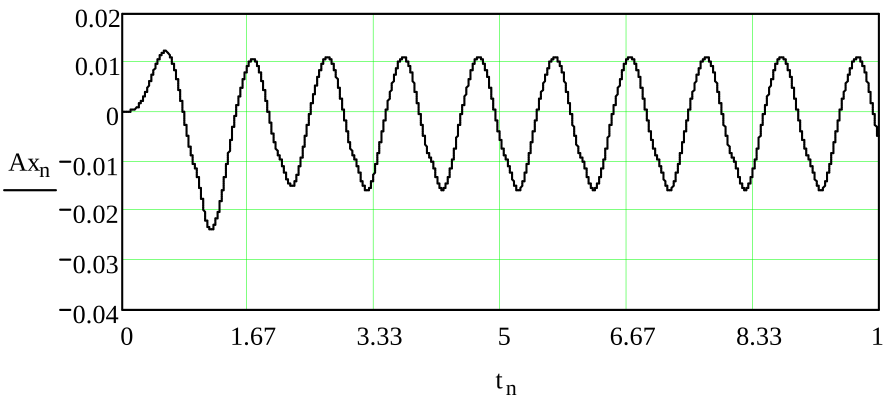


Fig. 9. Impulse *Ax(t)* in time domain (see equation (6)). Impulse is non-symmetric against zero level (non-symmetry is negative)

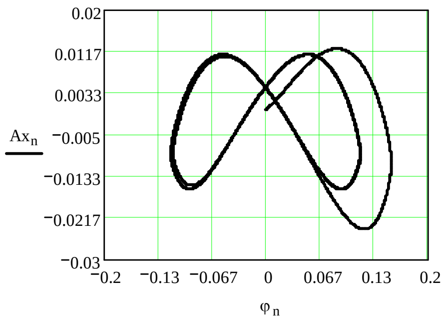


Fig. 10. Impulse *Ax(t)* as a function of angle φ. Impulse is non symmetric against zero level (non symmetry is negative)

$$IAx_n := \sum_n Ax_n$$

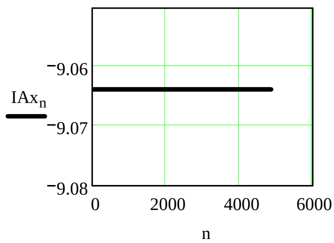


Fig. 11. Negative mean value of *Ax(t)* in time domain. Force of pivot push hull right side (what is necessary for robotics fish motions right side)

Synthesis of system with adaptive excitation and adaptive area control

In a case of adaptive excitation moment *M*, function of angular velocity in a form [3. 4] may be used:

$$M(t) = M0 \cdot sign(\omega).$$

Results of modeling are provided in Fig. 6. – 11. Comments about graphics are given under all Fig. 12. – 17.

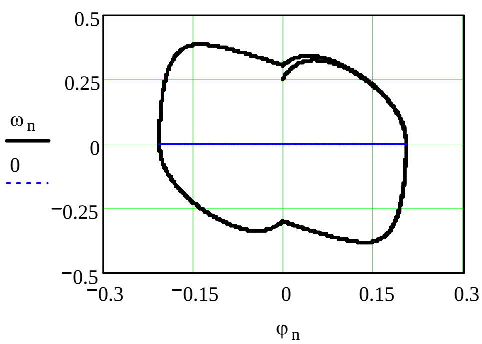


Fig. 12. Tail angular motion in phase plane – angular velocity as a function of angle. Because resistance of water is large transition process is very short. Adaptive excitation is more effective than harmonic excitation (see Fig. 6.)

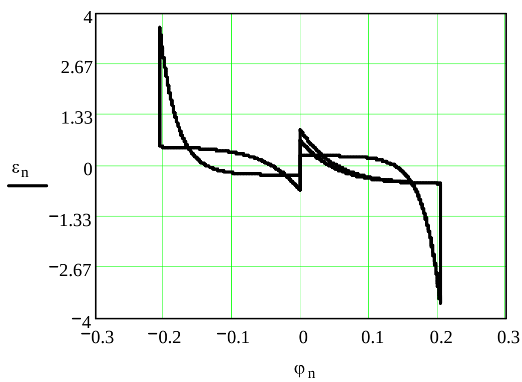


Fig. 13. Angular acceleration of tail as a function of angle. At the end of transition process graphic is more symmetric like graphic in Fig. 7

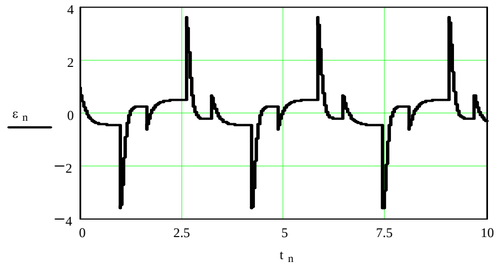


Fig. 14. Angular acceleration $\varepsilon = \ddot{\varphi}$ of tail in time domain (see equation (5)). Practically angular acceleration of tail reaches stationary cycle after half oscillation

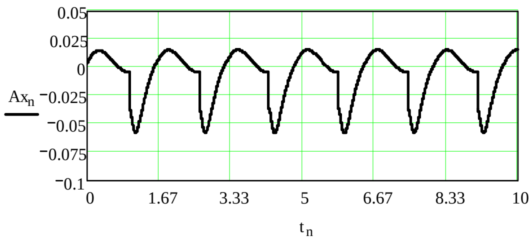
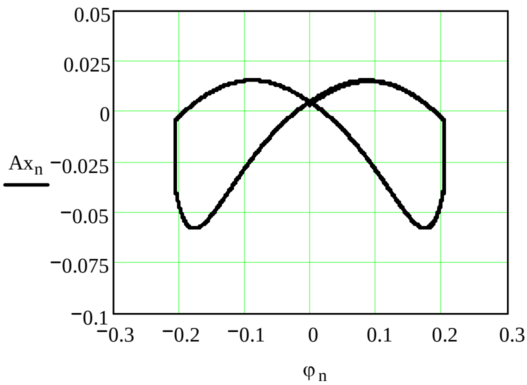


Fig. 15. Impulse *Ax(t)* in time domain (see equation (6)). Impulse is non-symmetric against zero level (non-symmetry is negative)



$$IAx_n := \sum_n Ax_n$$

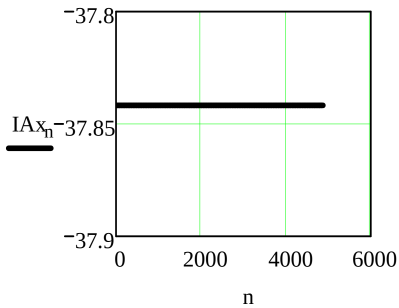


Fig. 16. Impulse *Ax(t)* as a function of angle φ. Impulse is non-symmetric against zero level (non-symmetry is negative). It means that criterion of optimization (2) is positive and force of pivot push hull right side

Fig. 17. Negative mean value of *Ax(t)* in time domain. Force of pivot push hull right side (what is necessary for robotics fish motions right side)

Conclusion

Robotic fish tail vibration based on simplified interaction with water flow provides easy analysis of equations of motion. This enables solution of the mathematical problem of area control optimization and provides information required for new system synthesis. Control (exchange) of object area under water allows generation of highly efficient mechatronic systems.

For realization of such systems adapters and controllers must be used. For this reason very simple control action have solutions with use of sign functions. Examples of synthesis of real mechatronic systems were given in the paper. The first example of synthesis is a system with time harmonic moment excitation of tail in the pivot. The second example is a system with adaptive force moment excitation as a function of phase coordinates. In both systems area exchange (from maximal to minimal values) has control action as a function of phase coordinates. It is demonstrated that real tail vibration motion is particularly stable and yields satisfactory real criterion in the hull contact point.

REFERENCES

[1] **E. Lavendelis.** Synthesis of optimal vibro machines. Zinatne, Riga, (1970). (in Russian).

[2] **E. Lavendelis and J. Viba.** Individuality of Optimal Synthesis of vibro impact systems. Book: Vibrotechnics”. Kaunas. Vol. 3 (20), (1973). (in Russian).

[3] **J. Viba**. Optimization and synthesis of vibro impact systems. Zinatne, Riga, (1988). (in Russian).

[4] **E. Lavendelis and J. Viba.** Methods of optimal synthesis of strongly non – linear (impact) systems. Scientific Proceedings of Riga Technical University. Mechanics. Volume 24. Riga (2007).

[5] http://en.wikipedia.org/wiki/**Lev_Pontryagin.** (2008).

[6] http://en.wikipedia.org/wiki/Pontryagin%27s_maximum_principle.(February (2008).

[7] http://en.wikipedia.org/wiki/Hamiltonian_%28control_theory%29. (2007).

[8] **V.G. Boltyanskii, R.V. Gamkrelidze and L. S. Pontryagin.** On the Theory of Optimum Processes (In Russian), *Dokl. AN SSSR*, 110, No. 1, 7-10 (1956).

[9] **L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze and E. F. Mischenko.** (Fizmatgiz). *The Mathematical Theory of Optimal Processes*, Moscow (1961).

[10] **L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko.** *The Mathematical Theory of Optimal Processes*, Wiley-Interscience, New York. (1962).

[11] **V. G. Boltyanskii.** *Mathematical Methods of Optimal Control*, Nauka, Moscow. (1966).

[12] **V. G. Boltyanskii.** Mathematical methods of optimal control: Authorized translation form the Russian. Translator K.N. Trirogoff. Balskrishnan-Neustadt series. New York. Holt, Rinehart and Winston. (1971).

[13] **E. B. Lee and L. Markus.** *Foundations of Optimal Control Theory*, Moscow: Nauka, (1972). (in Russian). 14. **Sonneborn, L., and F. Van Vleck.** (1965): *The Bang-Bang Principle for Linear Control Systems*, SIAM J. Control 2, 151-159.

[14] http://en.wikipedia.org/wiki/Bang-bang_control. (2008).