# 397. Rare events – rare attractors; formalization and examples

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**Abstract.** Analogy between attractors in nonlinear dynamics, called "rare attractors" by M.V. Zakrzhevsky and his colleagues [1] and emergencies, such as natural and technogenic catastrophes as well as downfalls caused by risky economic policies and strategies has been discussed. Examples of rare but technically significant attractors in nonlinear dynamics have been given.

**Keywords:** rare events, rare attractors, risks, vibration pendulum, synchronization.

## 1. INTRODUCTION

During the recent years the number of natural and man-induced disasters involving human casualties has dramatically increased [2]. In this connection, the challenge to be solved is the prediction of disasters and abatement of their consequences. This situation is broadly similar to dynamic phenomenon, called by M. V. Zakrzhevsky rare attractors [1]. Rare attractors can be also very important for applications in such cases. One of the purposes of this article is to accentuate this analogy.

It has been noted that the idea of using approaches of nonlinear dynamics in controlling the risks and in the problem of rare catastrophic events was stated in the report at the Presidium of the Russian Academy of Sciences [3].

## 2. ON THE ESTIMATION OF SIGNIFICANCE OF RARE EVENTS AND RARE ATTRACTORS IN VARIOUS AREAS OF HUMAN ACTIVITY

Estimation of significance factor of such events $Z$ can be considered as the product of "probability" of their occurrence $p$ by some measure of technical, economic or social significance of event $M$ :

$$Z = p \cdot M$$

It is so that an average man and sometimes even an expert intuitively estimates the significance of rare event.

In order to formalize this approach in every area, some definite methodologies have been developed for estimating both the probability of occurrence and the degree of significance [4, 5].

In case of emergencies the degree of significance is economic damage equivalent determined by special methods, and in case of estimating economic risks it is the loss of profit that is to be taken into account. In many cases in different areas it is often difficult to give reasonable estimation both of possible effect or damage and probability of situation. For example, in the problem of emergencies the significance factor can be dependent on "social risk", i.e. the attitudes of society towards different values of probability and damage and on subjective probabilities, reflecting the notion of danger of individual people or expert group. In order to account for these social and subjective factors sometimes coefficients or a set of multipliers can be introduced into the estimation formula.

It has been mentioned that identical situations are found in many other areas of human activity such as mining operations and mineral concentration (mining with low content of valuable component, search of rare nuggets), in gambling situations (in games with low probability of getting significant winning), in business (sale of expensive goods of low demand) and even in mathematics (studying the cases of strong influence of infinitesimal summand in equations).

The problem of controlling the rare attractors in nonlinear dynamics as well as in other areas is very important It consists in the widening areas of their existence and steadiness in parameter and phase space of the system in case of their utility and about narrowing these areas in case of undesirability.

## 3. ON THE SIGNIFICANCE OF SOME RARE ATTRACTORS IN THE VIBROENGINEERING

### 3.1. Pendulum with a vibrating axis of suspension (Stephenson – Kapitsa pendulum) [6, 7]

The classical condition of the local (Lyapunov) stability of the upper position of the pendulum with vertically vibrating axis of suspension (Fig. 1) is

$$A\omega > \sqrt{2gl} \qquad (1)$$

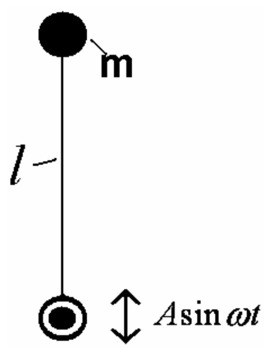


**Fig. 1.** Common pendulum (Stephenson – Kapitsa)

Here $A$ is the amplitude, $\omega$ – the frequency of vibration, $l$ – the length of the pendulum, $g$ – free fall acceleration. The inequality (1) can be considered as the condition of existence of the rare attractor.

But if we supply the pendulum with additional mass $m_1$ suspended on the spring $c$ (Fig. 2), than we obtain the following condition of stability (unlocal!) for the upper position of the pendulum [8, 9]:

$$A\omega > 0,6\sqrt{2gl} \qquad (2)$$

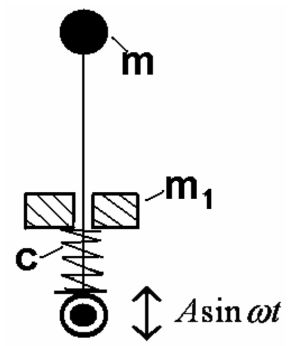


**Fig. 2.** Pendulum with the inner DOF

Thus, by introduction of additional DOF we obtain not only widening of the area of existence and stability but also unlocal stability of the upper position of the pendulum.

### 3.2. Multiple regimes of vibrational exciting and maintenance of rotation and multiple synchronization

The vibration of the axis of an unbalanced rotor with a certain frequency $\omega$ can maintain its stationary rotation with a mean angular velocity $<\dot{\varphi}> = \pm \omega p / q$, where p and q are whole positive numbers (Fig.3) [6, 7].

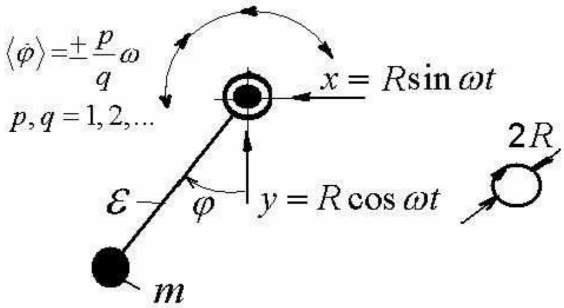


**Fig. 3.** Vibrational exciting and maintenance rotation of an unbalanced rotor

The same situation can be observed in the well known exercise "hoola-hoop" (Fig.4).

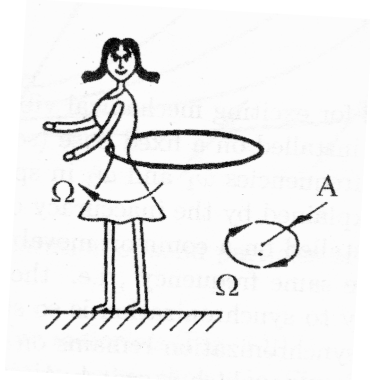


**Fig. 4.** The hoola-hoop exercise is the simple example of vibrational maintenance of rotation

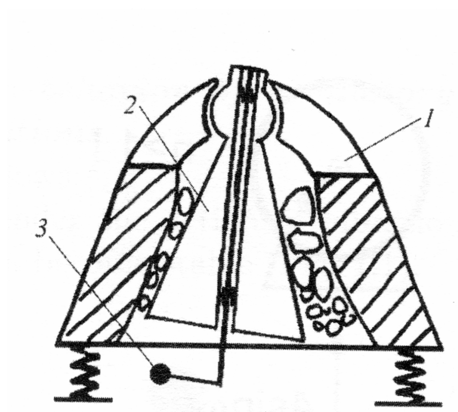


**Fig. 5.** Inertial crusher of the "Mekhanobr" Institute is an example of using the "hoola-hoop effect"
*(1 – body, 2 – inner cone, 3 – unbalanced rotor)*

For the main regime $p = q = 1$ we have the following condition of the maintenance of rotation:

$$N < N_{\max} = m\varepsilon\omega^2 R \,, \qquad (3)$$

where $N_{\max}$ is the maximal power which can be transferred by vibration to the shaft of the rotor, $m$ is the mass, $\varepsilon$ - the eccentricity of the rotor, $R$ – the amplitude radius of circular trajectory of vibration. The value $N_{\max}$ can be really very great. So, for instance, at $m\varepsilon = 10\, kg\, m$, $R = 2,5 mm$, $\omega = 314 s^{-1}$ we obtain $N_{\max} \approx 800\, kW$! This important phenomenon ensured the work of the inertial crusher (Fig. 5).

But for the multiple regime $p/q = 1/2$ when the mean angular velocity $< \dot{\varphi} >= \pm \frac{1}{2}\omega$, we obtain

$$N_{\max} = N^{(1/2)}{}_{\max} = \frac{2(m\varepsilon)^2 Rg\omega}{I} \,, \qquad (4)$$

where $I$ is the rotor moment of inertia in respect to axis of rotation. For the same values of parameters as above and $I = 1\, kg\, m^2$ we obtain

$$N^{(1/2)}{}_{\max} \approx 1,6\, kW \,,$$

i.e. approximately 500 times less than for the main regime. We have in this case the situation like that of a rare attractor. One of the ways of increasing the value $N^{(1/2)}{}_{\max}$ is to apply to the rotor a moment which periodically depends on the angle of rotation $\varphi$, for instance by using a spring, eccentrically connected to the rotor [6]. With such a spring (Fig.6) in the formula (4) instead of the value $mg\varepsilon$ there will be the value $mg\varepsilon + T_0 l$ where $T_0$ is the stress of the spring and $l$ is the distance from the pivot axis of the spring to the axis of the rotor. So, for instance if we assume that $T_0 = 60 mg\varepsilon$, we obtain

$$N^{(1/2)*}{}_{\max} = 61 N^{(1/2)} \cong 100\, kW \,.$$

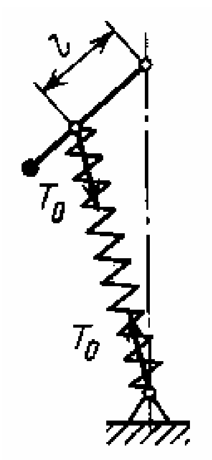


**Fig. 6.** One the ways of improving the condition of existence of the regime $p/q = 1/2$

In the problem of multiple self-synchronizatian of vibroexciters the above given way for the improving the condition of multiple regimes is also valid [6].

## 4. CONCLUSIONS

There are grounds for hope that the analogy between important emergency events and rare attractors in nonlinear dynamics described in this article will turn to be useful for both areas.

Particularly, the abilities of controlling emergency events and rare attractors deserve studying and development.

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## 5. REFERENCES


**[1]** **Zakrzhevsky M., Schukin I., Yevstignejev V.** Rare attractors in Driven Nonlinear Systems with Several Degrees of Freedom. – *Scientific Proceedings of Riga Technical University – Transport and Engineering,* serija 6, sejums 24, Riga, 2007, pp. 79-93.

**[2]** **Safonov O.** The nature began actively get rid of people. (www.utro.ru/articles/2008/07/03/749245/shtml).

**[3]** **Malinetsky G.G., Kurdyumov S.P.** Nonlinear Dynamics and Problems of Prognosis. – *Vestnik Rossiyskoi Academii nauk,* 2001, v. 71, № 3, pp. 210-232.

**[4]** **Vishnyakov Ja.D., Radaev N.N.** General theory of risks. – Moscow, Academia, 2007, 368 p.

**[5]** **Galatenko V.A.** Basics of information security. 4th issue – Moscow, INTUIT.RU, Binom, Labor. Of Sciences, 2008, 205 p.

**[6]** **Blekhman I.I.** Vibrational Mechanics. – World Scientific, Singapore at al, 2000, 509 p.

**[7]** **Ragulskis K.** Mechanisms on a vibrating foundation. – Problems of dynamics and stability. – *Ac. of Sc. Lietuva SSR,* Kaunas, 1963, 232 p.

**[8]** **Blekhman I., Sperling L.** Zum Einfluss eines inneren Freiheitsgrades auf das Verhalten eines Pendels mit vibrierender Aufhangung. – *Technische Mechanik,* 24, (2004b), 277-288.

**[9]** **Vasilkov V., Chubinsky A., Yakimova K.** The Stephenson-Kapitsa pendulum: Area of the Attraction of the Upper Positions of the Balance. – *Technische Mechanik,* 27, Half 1 (2007), 61-66.