379. THE NEW ANALYTICAL METHOD OF ACOUSTIC FIELD ESTIMATION IN THE ROOM

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Abstract. The directional characteristics of sound fields in enclosures are investigation in this paper. The main object of this paper is to determine the impulse response of sound sources for calculating acoustics parameters in closed space. The new analytical method has been suggested as the result of theoretical studies. This method allows a quantitative and qualitative analysis of the interaction of dynamic processes of a sound source with air in a room

Keywords: acoustic field, analytical method, estimation

Introduction

In an enclosed space, the propagation of sound should be described by particles or by waves. A model for sound wave propagation in a room leads to more or less efficient methods for solving the wave equation, for example using the Finite Element Method (FEM) and the Boundary Element Method (BEM) [1]. Another possibility is to describe the sound field in a room by sound particles moving around along sound rays. Such a geometrical model is using the simulation of sound in large rooms, for example the Ray tracing Method and the Image Source Method [2, 3]. This paper is intended to present new possibility to describe the propagation of sound in an enclosed space.

Description of the model

First is to create a model of a room, for example shown in Figure 1. The figure below shows a rectangular room. Within are air particles, shown as circles.

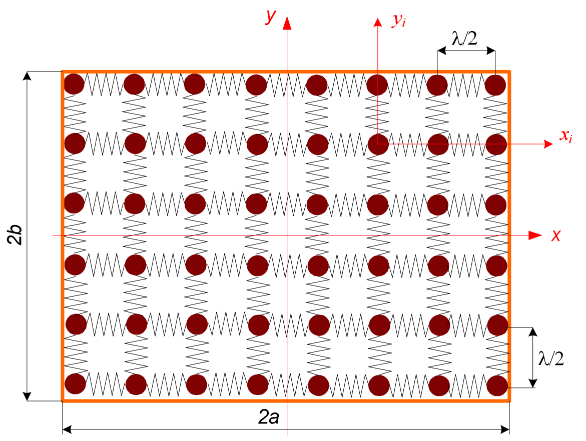

Fig. 1. The model of a room

The system of axes xy are fixed and axes x_i, y_i can translate with respect xy with speed of sound in air. So the motion of particles with respect to the frame x_i, y_i is relative. The number and location particles depend from known frequency and speed of sound in air, because:

$$\lambda_k = \frac{c}{\nu_k} \tag{1}$$

where
c – speed of sound in air;
ν_k – known frequency of sound wave;
λ_k – known wavelength.

The aim is to suggest the methodology allowing calculation of the relative displacement of air points under the action of the sound source in a known place in a room.

Mathematical model

Let's suppose that:

- walls of room is absolutely rigid and in equilibrium;
- the sound propagation is adiabatic;
- sound source characteristics and geometric measurements are known;
- the external volume (air mass) forces are ignored.

We shall choose a system of axes for the mathematical model and displacements (see Fig.2).

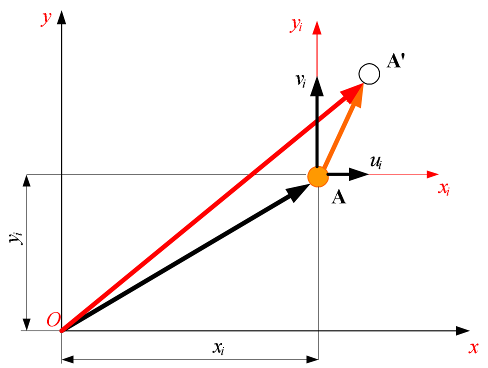

Fig. 2. Displacement counting diagram

Let's suppose that displacements towards Ax_i direction is u, towards Ay_i direction – v and we are going to find the function $\varphi(u,v,t)$, which describes displacements u, v for the boundary conditions:

- displacements at a wall are equal to zero, thus when $x=\pm a$, then displacements u=0, and when $y=\pm b$, then v=0 ;
- on a wall surfaces, the partial derivative of relative displacement by normal are equal to zero, i.e. $\frac{\partial\varphi}{\partial x}=0$, when $x=\pm a$; $\frac{\partial\varphi}{\partial y}=0$ when $y=\pm b$.

and in case of initial conditions:
$u=v=0$, displacements of the air points in the initial moment $t=0$ are equal to zero.
Then kinetic energy of air particles is expressed:

$$T_o=\frac{\rho_o}{2}\iint\limits_{(S)}\dot{\varphi}^2dxdy\ , \tag{2}$$

where

ρ_o — density of air;

S — integration area, i. e. the space part occupied by air.

Potential energy of external surface forces (sound source)

$$U_P=\int\limits_{(L)}\left(\bar{X}\varphi+\bar{Y}\varphi\right)dl\ , \tag{3}$$

where

$\bar{X},\bar{Y}$ – projections of external surface force (line unit is subjected to that force) on co-ordinate axes;

L — integration area, i. e. the surface part subjected to external surface forces.

Potential energy of air deformation forces

$$U_o=-\frac{\rho_o}{2c^2}\iint\limits_{(S)}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]dxdy \tag{4}$$

In that case Hamilton principle is expressed by equation:

$$\delta\int_{t_1}^{t_2}\left(T_o+U_P+U_o\right)dt=0\ . \tag{5}$$

By substituting expression (2) for (5), we get

$$\delta\int_{t_1}^{t_2}\left(\begin{array}{l}\frac{\rho_o}{2}\iint\limits_{(S)}\dot{\varphi}^2dxdy-\\ -\frac{\rho_o}{2c^2}\iint\limits_{(S)}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]dxdy+\\ +\int\limits_{(L)}\left(\bar{X}\varphi+\bar{Y}\varphi\right)dl\end{array}\right)dt=0\ . \tag{6}$$

Let's suppose that:

$$\varphi=\Phi q\ , \tag{7}$$

where

$\Phi=\Phi(x,y)$; $q=q(t)$.

The function Φ is selected on the basis of the boundary conditions, i. e. it should fit for the room presented in Figure 1.

Let's indicate the following

$$\frac{\partial\Phi}{\partial x}=\Phi_x\ ;\qquad \frac{\partial\Phi}{\partial y}=\Phi_y$$

Then variations u,v are the following:

$$\delta u=\frac{\partial\varphi}{\partial x}=\frac{\partial\Phi}{\partial x}q=\Phi_x q \tag{8}$$

and

$$\delta v=\frac{\partial\varphi}{\partial y}=\frac{\partial\Phi}{\partial y}q=\Phi_y q \tag{9}$$

By using equations (8), (9) and (7), we can express the potential energy of air deformation forces as:

$$U_o=-\frac{\rho_o}{2c^2}\iint\limits_{(S)}\left[\left(\Phi_x\right)^2+\left(\Phi_y\right)^2\right]q^2dxdy\ . \tag{10}$$

Having in mind the fact that variation is equated with derivative, i. e.

$$\delta q^2=2q\delta q\ , \tag{11}$$

variation of the potential energy of air deformation forces takes the following shape:

$$\delta U_o=-\frac{\rho_o}{c^2}\iint\limits_{(S)}\left[\left(\Phi_x\right)^2+\left(\Phi_y\right)^2\right]q\delta q dxdy \tag{12}$$

Derivatives of displacements with respect to time t :

$$\dot{\varphi}=\Phi\dot{q}\ . \tag{13}$$

Thus, we can rewrite expression of kinetic energy of air particles (2):

$$T_o=\frac{\rho_o}{2}\iint\limits_{(S)}\Phi^2\dot{q}^2dxdy\ . \tag{14}$$

Having in mind that

$$\delta\dot{q}^2=2\dot{q}\delta\dot{q}\ ,$$

variation of kinetic energy can be expressed:

$$\delta T_o = \rho_o \iint_{(S)} \Phi^2 \dot{q}\delta\dot{q}dxdy . \quad (15)$$

Variation of potential energy of external surface forces is the following:

$$\delta U_P = \int_{(L)} (\bar{X}\Phi + \bar{Y}\Phi)\delta q dl . \quad (16)$$

Thus, if we write computed expressions of variations of displacement (27) − (34) instead of displacements u,v in equations (2), (3) and (4) of mechanical energy T_o, U_P, U_o, we will obtain the same expressions of mechanical energy variations as in equalities (12), (15) and (16). So equation (6) can be rewritten

$$\delta\int_{t_1}^{t_2} \begin{pmatrix} \rho_o \iint_{(S)} \Phi^2 \dot{q}\delta\dot{q}dxdy - \\ -\frac{\rho_o}{c^2}\iint_{(S)} \left[(\Phi_x)^2 + (\Phi_y)^2 \right] q\delta q dxdy + \\ + \int_{(L)} \left(\bar{X}\Phi + \bar{Y}\Phi \right)\delta q dl \end{pmatrix} dt = 0 \quad (17)$$

We can integrate product $\dot{q}\delta\dot{q}$ by the method of partial integration:

$$\int_{t_1}^{t_2} \dot{q}\delta\dot{q}dt = \dot{q}\delta q \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \ddot{q}\delta q dt . \quad (18)$$

However, first member of right sides of equality in expression (18) is equal to zero, so

$$\int_{t_1}^{t_2} \dot{q}\delta\dot{q}dt = -\int_{t_1}^{t_2} \ddot{q}\delta q dt . \quad (19)$$

Thus, we obtain, that equality (17) resulting from Hamilton principle, is correct when having any value of δq and multiplier of δq equal to zero. In order to find function $q(t)$, we refer to equality (19) and obtain the following equation:

$$\begin{pmatrix} -\ddot{q}\rho_o \iint_{(S)} \Phi^2 dxdy - \\ -q\frac{\rho_o}{c^2}\iint_{(S)} \left[(\Phi_x)^2 + (\Phi_y)^2 \right] dxdy + \\ + \int_{(L)} \left(\bar{X}\Phi + \bar{Y}\Phi \right) dl \end{pmatrix} = 0 . \quad (20)$$

This equation can be rewritten:

$$m_o \frac{dq}{dt} = -k_o \int_0^{\tau} q dt + \int_0^{\tau} P dt , \quad (21)$$

where

$$m_o = \rho_o \iint_{(S)} \Phi^2 dxdy , \quad (22)$$

$$k_o = \frac{\rho_o}{c^2} \iint_{(S)} \left[(\Phi_x)^2 + (\Phi_y)^2 \right] dxdy \quad (23)$$

$$P = \int_{(L)} (\bar{X}\Phi + \bar{Y}\Phi) dl . \quad (24)$$

$$\tau = \frac{1}{4\nu} , \quad (25)$$

Equation (24) can be rewritten

$$P = \int_{(L)} \Phi(\int_0^{\tau} \bar{X}dt + \int_0^{\tau} \bar{Y}dt) dl \quad (26)$$

In a case of sound source, taking into account that the pressure of sound is the same in all directions $(\bar{X} = \bar{Y})$,

$$I = \int_0^{\tau} \bar{X}dt = \int_0^{\tau} \bar{Y}dt . \quad (27)$$

Then equation (26) can be rewritten:

$$\int_0^{t} P dt = \Phi_L I , \quad (28)$$

where

$$\Phi_L = \int_{(L)} \Phi dl . \quad (29)$$

Thus we obtain integral differential equation:

$$m_o \frac{dq}{dt} = -k_o \int_0^{t} q dt + \Phi_L I . \quad (30)$$

This integral differential equation can be solved approximately by means of the iteration method, for instance. Function $q(t)=t$ satisfying the initial condition $q(0)=0$ shall be regarded as a zero approximation. Then having inserted the chosen value in the right side of the equation (30) and then having integrated with the variable t, we get the first approximation:

$$q^{(1)} = \frac{\Phi_L I}{m_o} t - \frac{k_o}{6m_o} t^3 . \quad (31)$$

Having inserted the $q^{(1)}$ value on the right side of the equation (30) and having integrated, we get the second approximation:

$$q^{(2)} = \frac{\Phi_L I}{m_o} t - \frac{k_o \Phi_L I}{6m_o^2} t^3 + \frac{k_o^2}{120m_o^2} t^5 . \quad (32)$$

Analogically:

$$q^{(3)} = \frac{\Phi_L I}{m_o} t - \frac{k_o \Phi_L I}{6m_o^2} t^3 + \frac{k_o^2 \Phi_L I}{120m_o^3} t^5 - \frac{k_o^3}{5040m_o^3} t^7 . \quad (33)$$

$$q^{(4)} = \frac{\Phi_L I}{m_o} t - \frac{k_o \Phi_L I}{6m_o^2} t^3 + \frac{k_o^2 \Phi_L I}{120m_o^3} t^5 - \frac{k_o^3 \Phi_L I}{5040m_o^4} t^7 + \frac{k_o^4}{362880m_o^4} t^9 \quad (34)$$

Coefficients of the chosen function Φ can be found using Galiorkin method [4]:

$$\begin{cases} \int_{-a}^{a} dx \int_{-b}^{b} k_1 \varepsilon dy = 0 \\ \int_{-a}^{a} dx \int_{-b}^{b} k_2 \varepsilon dy = 0 , \\ \int_{-a}^{a} dx \int_{-b}^{b} k_3 \varepsilon dy = 0 \end{cases} \quad (35)$$

from the condition:

$$x = \pm a \text{ when } \frac{\partial \Phi}{\partial x} = 0 \text{ and } y = \pm b \text{ when } \frac{\partial \Phi}{\partial y} = 0 .$$

Knowing m_o, k_o, Φ_L and taking into account that sound intensity (pressure) is inverse proportional to the square distance from the point of sound source, we can calculate all four approaches for finding the approximate value q according to the equations (31) – (34). Finally, having applied equation (7), we can calculate approximate relative displacements of air particles:

$$u_i = \Phi_i q_{ui} ; \; v_i = \Phi_i q_{vi} . \quad (36)$$

Hereby presented, this theoretical study allows to solve approximately integral differential equation (30) and to calculate relative displacement of the material point of air under the action of the sound source.

Numerical examples

The problem simulated numerically is sketched in Fig. 1. For example, the geometrical values a=3.57 m and b= 3.21 m. Let's suppose that density of air $\rho_o = 1.224$ kg/m^3, speed of sound in air $c = 343$ m/s and function Φ :

$$\Phi = \frac{(x^2 - a^2)(y^2 - b^2)(x + b)}{(k_1 a^5 + k_2 a^4 x + k_3 b^3 y^2 + a^4 y + b^5) / a^{10}} \quad (37)$$

Table 1 shows the parameters for calculation of equations (3) –(6).

Table 1

Parameters	Value
k_1	-6,42799
k_2	6,05386
k_3	-0,876209
m_o	219,813
k_o	0,00115199

Fig. 3 and 4 illustrate fragments of the calculation results of acoustics field of frequency $\nu = 8000$ Hz when the sound source located at a different place of room.

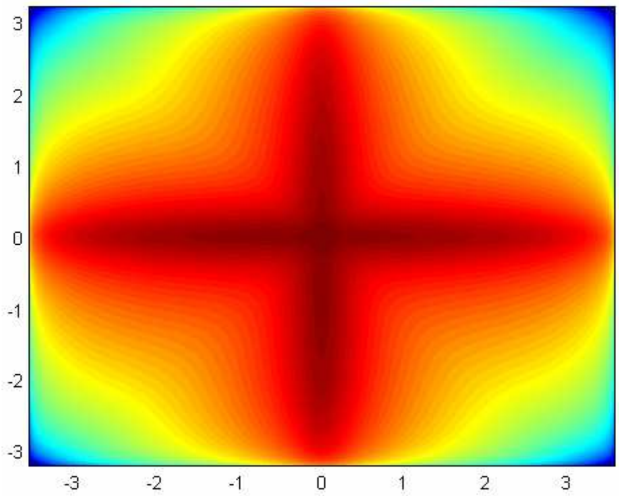

Fig. 3. Acoustics field when the sound source is located at the center of the room

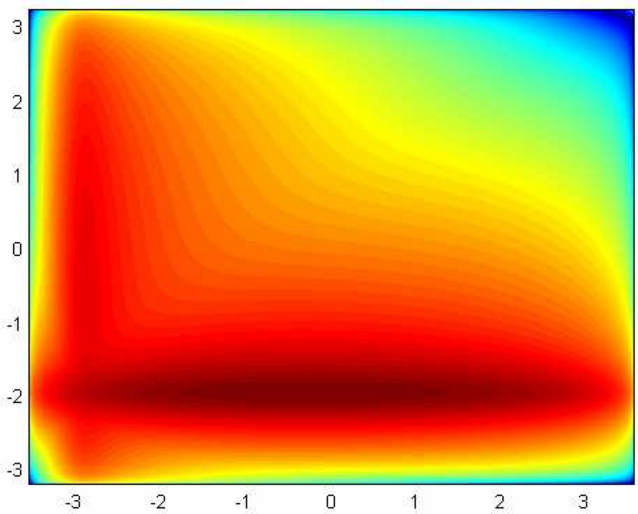

Fig. 4. Acoustics field when the sound source is located at $x = -3$ m and $y = -2$ m

Conclusions

The proposed analytical method allows the analysis of the interaction of dynamic processes of a sound source with air in a room and make possible to create precondition for operating acoustics fields in an enclosure.

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