350. Stable parametric identification of vibratory diagnostics objects

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Abstract. A common model of vibratory diagnostics objects is the stochastic difference schemes, and theirs parametrical identification is carried out least squares and least absolute deviations techniques. It is well known that these techniques are unstable under stochastic heterogeneity of observable process, specifically, in the presence of outliers. One way to make the stable parametrical identification of vibratory diagnostics objects is implementation of generalized least absolute deviations method based on concave loss function. Obtained requirements to the loss function guaranteeing the steadiness evaluation, algorithms of identification and examples are presented.

Keywords: autoregression; generalized least absolute deviations method; linear stochastic difference scheme; random vibration; stable evaluation of autoregression model factors; weighted least absolute deviations method.

Introduction

At present wide experience to create methods of vibratory diagnostic and rupture life forecast is accumulated. One of exploitable mathematical model for these problems is linear stochastic difference scheme

$$y_k - \sum_{j=1}^{L} a_j y_{k-j} = \xi_k, \quad k = L+1, L+2, \ldots \qquad (1)$$

in which $y_l = y(l\Delta)$, $l = 0, 1, 2, 3, \ldots,$ are data vibration at point of time $l\Delta$; Δ is sampling interval; $\{\xi_k\}$ are unobservable stochastic processes; a_j, $j = 1, 2,\ldots, L$ are the model parameters appointed at the design stage; L is the size of log. It is significant that unobservable random values ξ_k are in accepting independent values under diagnostic problems.

For example it is the established fact [1] that random vibration of single mass linear mechanical system satisfies Eq.(1) under $L = 2$, i.e. it is second-order autoregression process

$$y_k = a_1 y_{k-1} + a_2 y_{k-2} + \xi_k, \quad k = 3, 4, 5, \ldots \qquad (2)$$

Autoregression factors a_1, a_2 of this process are unambiguously interdependent with resonance frequency and damping decrement of the system i.e.

$$f_0 = \frac{1}{2\pi\Delta}\arccos\frac{a_1}{2\sqrt{-a_2}}; \quad \delta = -\frac{1}{2 f_0 \Delta}\ln(-a_2)\,. \qquad (3)$$

Object design characteristics and interdependent model Eq. (1) factors a_j, $j = 1, 2,\ldots, L$ are denatured under development of structure degradation processes. System malfunction may be diagnosed by variations with time of coefficients a_j being estimated by data vibration signals y_k.

To decrease false alarm and aim omission risks it is issue of the day that the problem is taking into account overidentification and heterogeneousness of data vibration signals under estimating of coefficients a_j. Sources of overidentification and heterogeneousness of data vibration are a) part of sampling may be mismatch to the accepted model because of embryonic defect; b) instability of error variance of measuring; c) availability of outliers in the middle of y_k; d) multiplicative nature of noises ξ_k.

It is well known that the least squares and least absolute deviations techniques (LST and LADT) are unstable under stochastic heterogeneity of observable processes, specifically, in the presence of outliers [2] . One way of doing the stable parametrical identification of vibratory diagnostics objects is implementation of generalized least absolute deviations technique (GLADT) based on concave loss function [3] .

Generalized least absolute deviations method

Follow [3] we define GLADT estimation of parameters a_j, $j = 1, 2,\ldots, L$ for model (1) and data $\{y_k : k = 1, 2, 3, \ldots, n\}$ as

$$\bar{\mathbf{a}} = (\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_L) = \arg\min_{\mathbf{a}\in\mathbf{R}^L} \sum_{k=L+1}^{n} \rho\left(\left| y_k - \sum\nolimits_{l=1}^{L} a_l y_{k-l} \right|\right), \quad (4)$$

where function $\rho(x)$ is monotone increasing and twice differentiable for all nonnegative x, $\rho(0) = 0$, and $\rho''(x) \le 0$.

It is proved [3] that all local minimums of the goal function of problem (4) form set:

$$U = \left\{ \left(a_1^{(\mathbf{k})}, a_2^{(\mathbf{k})}, ..., a_L^{(\mathbf{k})}\right) : y_k = \sum_{l=1}^{L} a_l^{(\mathbf{k})} y_{k-l}; \right.$$
$$\left. k \in \mathbf{k} = \{k_0, k_1, ..., k_L : 1 \le k_0 < k_1 < ... < k_L \le n \} \right\}. \quad (5)$$

Hence we may look for $\overline{\mathbf{a}}$ by means of solving $\mathbf{C}_n^L$ systems of L linear equations and choosing optimal vector from U. Under L = 1, 2, 3 such enumeration of possibilities is realizable.

Let us consider interrelation between GLADT and weight least absolute deviations technique (WLADT).

Statement 1.

$$\left(\forall \{ p_k \ge 0 : k = L+1, L+2, \dots, n\}\right)\left(\arg\min_{\mathbf{a}\in\mathbf{R}^L} \sum_{k=L+1}^{n} p_k \left| y_k - \sum_{l=1}^{L} a_l y_{k-l} \right| \in U \right). \quad (6)$$

Proof. As is easy to see problem (6) is optimization of the piecewise linear convex function. The introduction of slack variables leads us to linear programming problem

$$\min_{\substack{a_1,\dots,a_L \\ u_{L+1},\dots,u_n}} \left\{ \sum_{k=L+1}^{n} p_k \cdot u_k : \quad -u_k \le y_k - \sum_{l=1}^{L} a_l y_{k-l} \le u_k, u_k \ge 0, k = L+1, 2, \dots, n \right\}. \quad (7)$$

Task (7) has the canonical form, $n - 1$ variables and $3(n - L - 1)$ constraints including the nonnegativity constraints for variables u_k. Let m be equal to the number of zero values of variables u_k at optimal solution of the task, i.e. exactly m conditions

$$y_k = \sum_{l=1}^{L} a_l y_{k-l}$$

are held. Then active constraints are m nonnegativity constraints for zero variables u_k, m common constraints with this variables u_k, and $n - m - L - 1$ constraints for positive variables u_k. General number of the active constraints equals to $n + m - L - 1$. On the other hand it is necessary number of active constraints for the optimal basic solution no less than $n - 1$. Therefore we have $m \ge L$.

Statement 1 is proved.

Statement 2.

$$\left(\forall (a_1^{(\mathbf{k})}, a_2^{(\mathbf{k})}, \dots, a_L^{(\mathbf{k})}) \in U\right)\left(\exists \{ p_k \ge 0 : k = L+1, L+2, \dots, n\}\right):$$
$$(a_1^{(\mathbf{k})}, a_2^{(\mathbf{k})}, \dots, a_L^{(\mathbf{k})}) = \arg\min_{\mathbf{a}\in\mathbf{R}^L} \sum_{k=L+1}^{n} p_k \left| y_k - \sum_{l=1}^{L} a_l y_{k-l} \right|. \quad (8)$$

Proof. Let $\chi_{\mathbf{k}}(*)$ be characteristic function for set $\mathbf{k}$ (5). The set $\left\{ p_i = \chi_{\mathbf{k}}(i) : i = L+1, 2, \dots, n \right\}$ is hold condition of the statement for given $\mathbf{k}$.

Statement 2 is proved.

Ascertained interrelation between GLADT and WLADT permits to ground GLADT estimation for stable evaluation of autoregression model factors under availability of outliers in the middle of y_k. Groundlessness of LADT estimation under such conditions is proved in [2] In the same place existence of stable WLADT estimation is ascertained, but general way to look of proper weights $\{ p_k : k = L+1, 2, \dots, n\}$ is not presented.

Further we describe the implementation of GLADT for stable evaluation of autoregression model factors. Like in [2] we apply the influence functional presented in [4] .

Stability of evaluation of autoregression model factors

Let $\mathbf{x}_n = (x_{1-p}, \dots, x_n)$ be stationary in the wide sense time series. We observe autoregression model

$$\text{AR}(L): \quad x_k = \sum_{i=1}^{L} a_i x_{k-i} + \varepsilon_k, \; k \in \mathbf{Z},$$

where L is known order, $\mathbf{a} = (a_1, \dots, a_p)$ is vector of nonrandom factors, $\{\varepsilon_k\}$ are independent identically distributed variates with non-degenerate distribution function. Let values

$$y_k = x_k + z_k^{\gamma} \xi_k, \; k \in \mathbf{Z}$$

be observable variables, $\{z^{\gamma}_k\}$ be independent identically distributed variates, $\{z^{\gamma}_k\} \sim \text{Bi}(1, \gamma)$, $0 \le \gamma \le 1$, γ be obstruction level, $\{\xi_k\}$ be independent identically distributed variates with distribution μ_ξ from class M_ξ; successions $\{x_k\}$, $\{z^{\gamma}_k\}$, $\{\xi_k\}$ be independence. In that way we observe simple obstruction scheme of data by independence outliers.

Under appearance obstruction (5) traditional estimations are inconsistent. For measuring of quality of estimation $\widehat{\mathbf{a}}_n$ for vector $\mathbf{a}$ under observable data $\mathbf{y}_n$ we suppose existence of convergence in probability $\widehat{\mathbf{a}}_n \xrightarrow{\mathbf{P}} \mathbf{a}_\gamma$ and equality $\mathbf{a}_0 = \mathbf{a}$.

Simple infinitesimal characteristic of estimation $\widehat{\mathbf{a}}_n$ stability under data obstruction $\{y_k\}$ is vector

$$IF(\mathbf{a}_\gamma, \mu_\xi) = \lim_{\gamma \to 0} \frac{\mathbf{a}_\gamma - \mathbf{a}_0}{\gamma}$$

named as influence functional for estimate $\widehat{\mathbf{a}}_n$ [4] This functional characterizes the value of main linear member of asymptotic expansion of displacement

$$\mathbf{a}_\gamma - \mathbf{a}_0 = IF(\mathbf{a}_\gamma, \mu_\xi)\gamma + o(\gamma). \quad (9)$$

Condition of estimation stable is finite sensitivity to great mistake

$$GES(M_\xi, \mathbf{a}_\gamma) = \sup_{\mu_\xi \in M_\xi} \left| IF(\mathbf{a}_\gamma, \mu_\xi) \right| < \infty.$$

In that case the main linear member of asymptotic expansion (7) is uniformly small for all obstructions and small γ.

Let GLADT estimation of parameters $\hat{\mathbf{a}}_n^{GLM}$ be defined by way of algorithm (4). Qualitative assessment of stability for GLADT estimation $\hat{\mathbf{a}}_n^{GLM}$ under outliers gives the following theorem.

Theorem. *Let time series (5) be observed. If loss function $\rho(x)$ of algorithm (4) is so that* $\sup_{x\ge 0}\left|x^2\rho'(x)\right|<\infty$ *then* $GES(\mathrm{M}_\xi,\mathbf{a}_\gamma^{GLM})<\infty$.

Proof. The necessary conditions of minimum for task (4) are bridging set

$$\sum_{k=1}^{n}\rho'_{a_l}(\varepsilon_k)\,y_{k-l}\,\mathrm{sign}\left(y_k-\sum_{i=1}^{L}a_i y_{k-i}\right)\div 0,\quad l=1,\dots,L \qquad (10)$$

It is proved in [2] that WLADT estimation obtained by the way solving of bridging set

$$\sum_{k=1}^{n}p(y_{k-1},\dots,y_{k-1}\mathrm{sign}(y_k-\sum_{i=1}^{L}a_i y_{k-i})\div 0, \qquad (11)$$
$$l=1,\dots,L$$

under condition $\sup_{y\in\mathbf{R}^p}\left|\mathbf{y}\mathbf{y}^T p(\mathbf{y})\right|<\infty$ has finite sensitivity to great outliers, i.e.

$$GES(\mathrm{M}_\xi,\mathbf{a}_\gamma^{GLM})<\infty.$$

Comparison of Eq. (10) and Eq. (11) as well as taking into account proved statements 1 and 2 implies assertion of the theorem under

$$\sup_{x\ge 0}\left|x^2\rho'(x)\right|<\infty.$$

Theorem is proved.

As is easy to see the concave increasing functions $1-\exp(-|x|)$; $\arctan|x|$; $|x|/(1+|x|)$ are holding the theorem conditions, and functions $x^2; |x|; \sqrt{x}; \ln(1+|x|)$ are not.

Computational experiments

Process AR(1) with one-sided outliers

$$\begin{cases} x_0=0,\\ x_k=\alpha x_{k-1}+\varepsilon_k,\ \ k=0,1,\dots,M.\\ y_k=x_k+z_k^{\gamma}\xi_k,\end{cases}$$

Here $\alpha=0.7$; $\varepsilon_k\sim N(0,\sigma_{zk}^2)$; σ_{zk}^2 are variates distributed uniformly in segment [0;2]; $\{z^{\gamma}_k\}$ are independent identically distributed variates; $\{z^{\gamma}_k\}\sim\mathrm{Bi}(1,\gamma)$, $0\le\gamma\le 1$, γ are an obstruction level; $\{\xi_k\}$ are independent variates distributed uniformly in segment [50; 100]; M =1500 is the number of trials.

Test computer simulation is identification of autoregression factor α at sight of signals y_k evaluated according to Eq. (8). Identification algorithms are GLADT estimation (4) with the difference loss functions.

Simulation data is shown in Fig. 1.

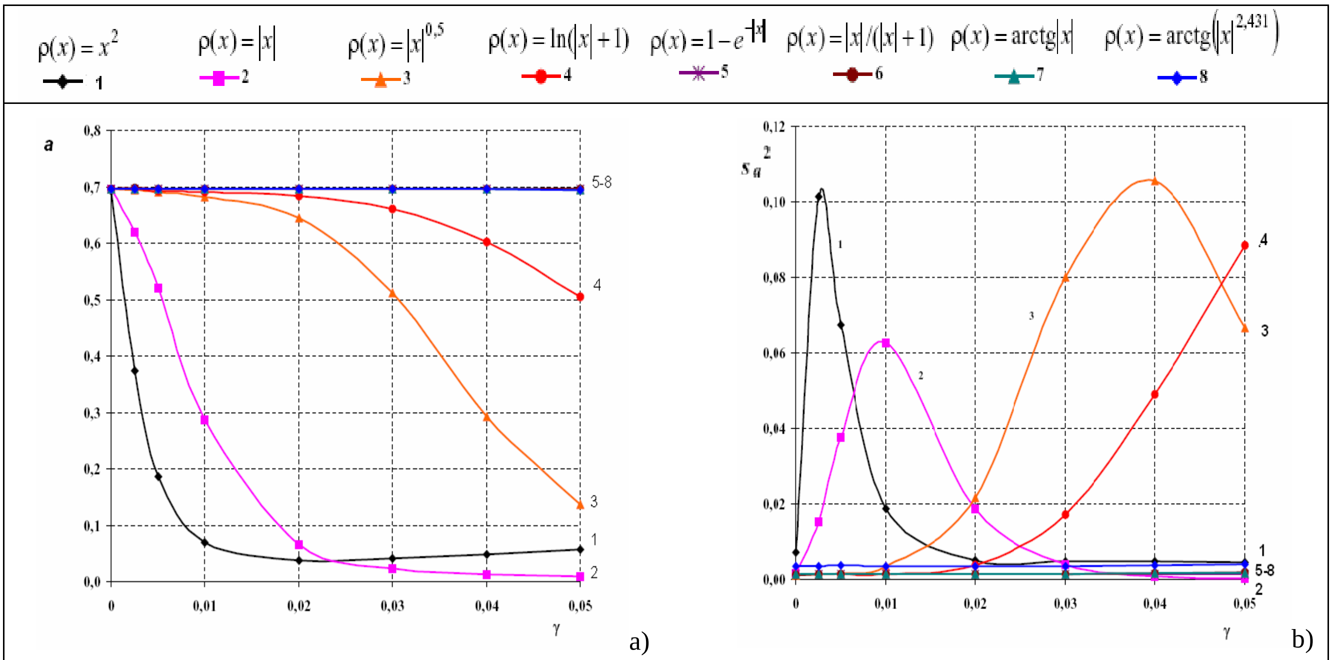

Fig. 1. Process AR(1) simulation data: a) LST, LADT and GLADT estimations of autoregression factor α; b) Sample variance s_a^2 of the estimations

Fig. 1. a demonstrates that LST and LADT estimations are inconsistent in spite of the fact that sample variance of these estimations is small (see Fig. 1.b). GLADT estimations with the loss functions satisfying theorem condition (lines 5, 6, 7, and 8) are consistent and have small sample variance of ones.

Vibrations of single mass linear mechanical system with resonance frequency $f_0 = 91.7$Hz and damping decrement $\delta = 1.22$ under sampling interval $\Delta = 0.001$sec satisfies second-order autoregression process (2) with factors $a_1 = 1.5$, and $a_2 = 0.8$. Simulating vibration of this system we evaluate by AR(2) process

$$\begin{cases} x_0 = x_1 = 0, \\ x_k = 1,5x_{k-1} - 0,8x_{k-2} + \varepsilon_k, \qquad k = 0,1,\ldots,M, \\ y_k = x_k + z_k^{\gamma}\xi_k, \end{cases} \qquad (12)$$

Here $\varepsilon_k \sim N(0,\sigma_{zk}^2)$; σ_{zk}^2 are variates distributed uniformly in segment [0;2]; $\{z^{\gamma}_k\}$ are independent identically distributed variates; $\{z^{\gamma}_k\} \sim \mathrm{Bi}(1, \gamma)$, $0 \le \gamma \le 1$, γ are an obstruction level; $\{\xi_k\}$ are independent variates distributed uniformly in segment [50; 100]; $M = 100$ is number of trials.

Test computer simulation is identification of autoregression factors a_1, and a_2. at sight of signals y_k evaluated according formulas (12). Identification algorithms are GLADT estimation (4) with the difference loss functions.

Simulation data is shown in Fig. 2.

Again we observe that GLADT estimations with loss function satisfying the theorem condition are consistent only.

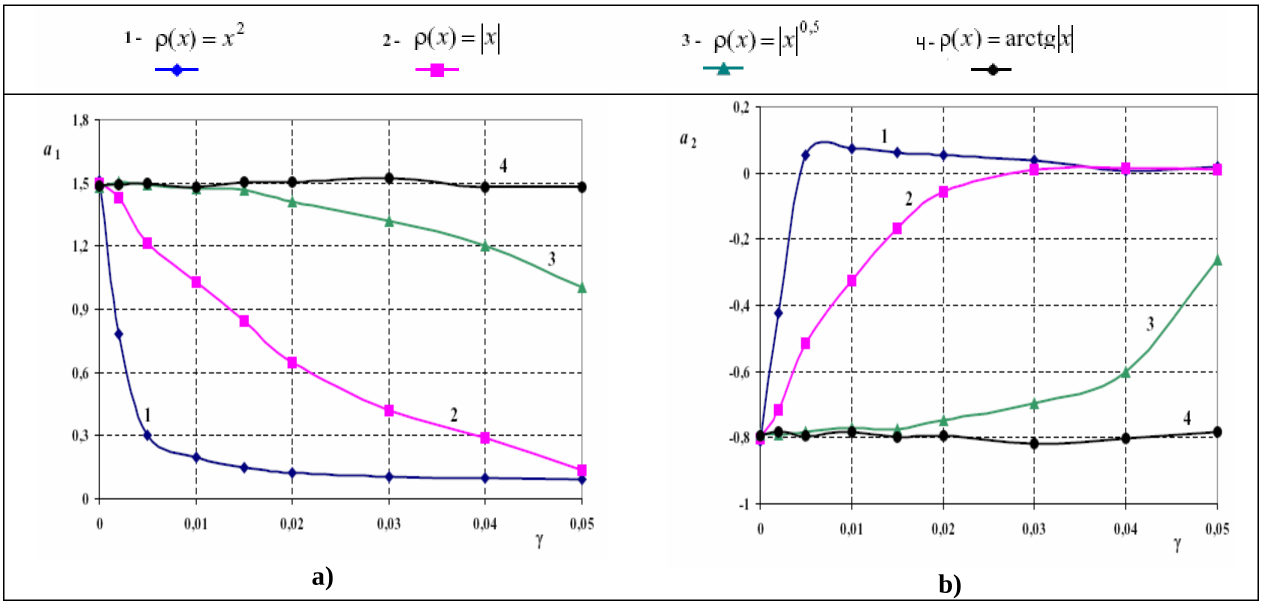

Fig. 2. Single mass linear mechanical system simulation data: a) LST, LADT and GLADT estimations of autoregression factor a_1; b) LST, LADT and GLADT estimations of autoregression factor a_2

Table 1. Results of tests

□	f_0				□			
	ρ(x)=x²	ρ(x)=|x|	ρ(x)=|x|^0.5	ρ(x)=atan|x|	ρ(x)=x²	ρ(x)=|x|	ρ(x)=|x|^0.5	ρ(x)=atan|x|
0	91,7976	92,8006	93,0717	93,1643	1,1529	1,1873	1,2738	1,2401
0,002	146,8953	90,5264	89,3832	90,1766	2,9343	1,8355	1,3090	1,3600
0,005		88,7691	90,2000	91,1608		3,7423	1,3433	1,2535
0,01		70,9415	92,1783	92,1390		7,9464	1,4036	1,3204
0,015			93,1951	90,9585			1,3781	1,2432
0,02			98,0430	90,0422			1,4894	1,2760
0,03			105,0666	91,6342			1,7251	1,1270
0,04			109,1621	94,3107			2,3368	1,1538
0,05			30,3857	92,0200			21,9591	1,3124

As indicated above (see Eq. (3)) autoregression factors a_1, a_2 of this process are unambiguously interdependent with resonance frequency and damping decrement of system. Estimation of resonance frequency f_0 and damping decrement δ evaluated through autoregression factors (see Eq. (3)) are presented in Table 1. Empty strings in this table signify impossibility of the target calculation under current obstruction level γ.

As may be seen from Table 1 the best technique in the absence of obstructions ($\gamma = 0$) is LST. But application of LST staves off calculations under $\gamma = 0.005$. On the other hand the application of GLADT estimation with the loss functions satisfying to the theorem condition enables to make stable estimations.

Summary

As the statements indicates that adduced theoretical proofs and computer simulation demonstrate possibility of stable parametric identification of vibratory diagnostic objects by usage of GLADT estimation with loss the function satisfying to the condition of the proved theorem.

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