328. Concept and Methods of Adaptive Vibration Protection and Stabilization

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Abstract. The paper considers the concept, the general formulation of problems and the algorithms of adaptive real-time control of multivariate vibration protection and stabilization of multi-axis chassis position in space, which provide in general case the solution of the problems of oscillations moderation and motion stability.

Keywords: Adaptive control, vibration protection, adaptive stabilization, adaptive regulator, stability, multi-axis machines

Introduction


Further essential improvement of quality of these systems is obtained based on concepts of the inverse problems of controlled system dynamics [2, 5] and adaptive regulators of a new generation [12-14], which combine accuracy and high speed without overshoot. The original approach and methods provide in a general case adaptive vibration protection and stabilization in real time, taking into account the motion stability.

Mathematical models

The position in space for the case of the multi-axis machine with essentially nonlinear passive suspension, functioning together with an active suspension, is characterized by the phase coordinates $z, \phi, \psi$ (the vertical displacement of the centre of masses, the angle of pitch and the angle of roll for the case of the multi-axis machines, for which the command values $z_5, \phi_5, \psi_5$ are assigned). The motion equations of the controlled system $u = (u_5, \ldots, u_8)^T$ are assumed:

$$\ddot{x} = F(\dot{x}, x, u),$$

$t \geq t_0 : x(t_0) = x_0, \dot{x}(t_0) = \dot{x}_0,$

where $F = (F_1, \ldots, F_8)^T$ is a vector of the right-hand side of the system; $x = (x_1, \ldots, x_8)^T$ is a vector of the generalized coordinates; $\dot{x} = (\dot{x}_1, \ldots, \dot{x}_8)^T$ is a vector of the generalized speeds; $x_o = (x_{o1}, \ldots, x_{o8})^T$ is a vector of initial conditions; $x_0 = (x_{01}, \ldots, x_{08})^T$ is a vector of initial speeds; $T$ is the symbol of transposing.

In the system (1) the motion equations of the electrohydraulic drives are considered in the following form:

$$\dot{q}_5 = \phi_5(q_5, u, q_6);$$
$$\dot{q}_6 = \phi_6(q_6, q_5, q_2);$$
$$\dot{q}_7 = \phi_7(q_7, q_2);$$
$$\dot{q}_8 = \phi_8(q_8, \dot{q}_2);$$

$t \geq t_0 : q_i(t_0) = q_{i0}, i = 5, \ldots, 8,$

where $\phi_5 \sim \phi_8$ are known vector operators.

Servo-drives can be installed on all the supports, or only on the required ones (for example, only on the side supports). There are unified quality criteria for all the drives, which are presented in the form of desirable motion properties, of formulation of the problems of control and restriction, as well as in the form of adaptive control algorithms, which parameters, however, vary for the drives.
of the various supports. For the chassis with pneumatic elastic elements it is natural to control the position of the plunger of the pneumatic elastic element by controlling the submission of the liquid in the cavities above and under the plunger.

In the mathematical model of the spatial oscillations of the machine (1), the virtual elastic elements (led to the supports) are examined. At this, for an active suspension the deformation of the $i$th support, which is led to the wheel/roller, is defined by the expression:

$$\Delta_i(t)=y_i(t)-z_i(t)+q_{6i}(t) \quad (i=1, n; \ l=1, 2),$$  \hspace{1cm} (2)

Where $z_i(t) = z(t) + l_1 \varphi_1 + b_1 \psi_1(t)$; $y_i$ are the displacements of the non-amortized masses; $q_{6i}$ are plunger displacements; $l_1, b_1$ are geometrical parameters.

The virtual displacements of the plungers of the actuating servo-cylinders have the following meaning:

$$q_{6i} = \beta_{li} q_{6i},$$

where $\beta_{li}$ is a transfer function of reduction of the physical characteristic to the virtual: $\beta_{li} = \beta_{li}(t)$; $q_{6i}$ are the displacements of the plungers of the physical servo-drives. The virtual variables are recalculated into the physical variables (displacements, speeds and accelerations on the real drive), taking into account the factors of reduction, while designing.

**Observables variables**

The accelerations of the amortized and non-amortized masses and the displacements of plungers of the actuating mechanisms can be considered as the observable variables. In the other variant, the accelerations of the amortized mass, displacements of the plungers of actuating mechanisms and dynamic loadings on the road (or their derivatives) can be observed. All the other variables, used by the smart regulators, can be identified.

Let the accelerations of the amortized mass above the corresponding supports $\ddot{z}_i (i=1, n; \ l=1, 2)$, accelerations of the non-amortized masses $\ddot{y}_i (i=1, n; \ l=1, 2)$ and displacements of the plungers of actuating mechanisms $q_{6i} (i=1, n; \ l=1, 2)$ be observable. In this case, $4n$ accelerations sensors and $2n$ displacements sensors are required. Values, measured by sensors, are estimations of the observable variables. The variables that are missing for control formation can be identified later on.

The desirable condition for the case of the machine is assigned by the command values of the variables $z, \varphi, \psi$. The command values of the control variables for each channel of control are calculated according to the formula:

$$\Sigma_i = z_i + l_1 \varphi_i + b_1 \psi_i \quad (i=1, n; \ l=1, 2).$$  \hspace{1cm} (3)

Apparently, the command values of the control variables for each support in a multivariate control system represent function of the assigned parameters of stabilization of the case of the machine $z, \varphi, \psi$ and also of the parameters $l_1, b_1$, which describe the servo-drives position, referring to the centre of masses of the vehicle.

The quality criteria for each of the drives differ by the command values (3) for each of them, by the current values of the phase variables, and also by the control parameters. Regulators generate control signals for each servo-drive while functioning:

$$u_i = \Phi(z_i, \varphi_i, \psi_i, z_{di}, f) \quad (i=1, n; \ l=1, 2).$$

where $\Phi(.)$ is a known operator [12-14]. Each servo-drive controls the spatial condition of the virtual mass $m_{di}$, which is a part of the general amortized mass, connected to the $i$th support.

An active suspension of the chassis at each moment of time must provide the following:

- minimization of the oscillations of the amortized mass,
- stabilization of the dynamic loadings on the controlled wheels in relation to the corresponding static loadings.

**Conditions of stability**

Besides the general conditions of motion stability, according to Lyapunov, which are provided at synthesis of the control functions, the requirement of stabilization of dynamic loadings on control wheels, in relation to the corresponding static loadings, adequate to the requirement of minimization of the take-off of the controlled wheels of the machine from the ground, corresponds to the supplying of the machine with the motion stability.

In the problem of active moderation of the oscillations, the output variable is the vertical displacement of the amortized mass $x(t)$, and in the problem of stability the output variable is the vertical loadings on controlled wheels of the machine $P_{i4}(t) = P_{4i}(t) + P_{3i}(t)$.

The take-off of the tires from the ground occurs when the following condition is satisfied:

$$P_{4i}(t) \leq -\bar{P}_{4i} = -(m_{4i} + m_{2i})g$$  \hspace{1cm} (5)

or

$$\delta_{4i}(t) \leq \delta_{4i} = (m_{4i} + m_{2i})g / C_{si},$$

where $\delta_{4i}(t) = q_{4i}(t) - y_{4i}(t)$; $\delta_{4i}$ is static deformation of the $i$th tires; $q_{4i}(t)$ is the kinematical perturbation; $C_s$ is the rigidity of the $i$th tire, which corresponds to the small oscillations.

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When fluctuations are viewed with respect to the position of static equilibrium of system, both problems belong to a class of vibration protection tasks, when the command value of an output variable is assigned to zero, and to the stabilization tasks if the output variable is not equal to zero.

The algorithm of the solution of a stability problem of controlled wheels is defined by the properties of the used sensors. We will consider two alternatives.

1. The accelerations of the amortized and non-amortized masses are observed (measured) and the current values of dynamic loadings \( P_i(t) = m_{il} \ddot{x}_i + m_{2il} \ddot{y}_i \) are calculated.

Then the derivative is defined by differentiation:
\[
P(t) = m_1 \ddot{x} + m_2 \ddot{y}
\]

2. The dynamic loadings within the contact of a tire with the road are observed: \( P_i(t) = P_{4il}(t) + P_{5il}(t) \). After differentiation we obtain its derivative \( P_i(t) = dP_i(t)/dt \). If the derivatives of dynamic loadings \( \dot{P}_{4il}(t) = \dot{P}_{4il}(t) + \dot{P}_{5il}(t) \) are observed, we can find the force by integration: \( P_0(t) = \int \dot{P}_i(t) dt \) (at zero initial conditions).

**Formulation of the problem**

The problem of adaptive control of active suspension consists of the solution of two problems at each moment of time:
- minimization of the oscillations of the amortized mass;
- maintenance of the motion stability (minimization of the take-off of the controlled wheels from the ground).

Let's consider formulations of these problems, leaving out the indexes of supports and boards of the machine for simplicity.

**Formulation of the vibration protection problem**

For object (1) for the output variable \( x(t) \) the desirable motion properties are set in the following form:
\[
x = f(\bar{x}, \dot{x}, x),
\]

\( t \geq t_o; x(t_o) = \bar{x}_o, \dot{x}(t_o) = \dot{x}_o \),

where \( f(\cdot) \) is the set operator, generally nonlinear, defined on fuzzy sets; \( \bar{x} \) is the command value (in the problem of vibration protection \( \bar{x} = 0 \)) and the condition of asymptotical stability looks as follows:
\[
x(t) \rightarrow \bar{x}(t), \dot{x}(t) \rightarrow 0 \text{ at } t \rightarrow \infty.
\]

It is necessary to construct a regulator, which provides fulfillment of the conditions of the motion optimality (6) and asymptotical stability (7).

In a specific case, the desirable motion properties of an output variable are assigned by a linear equation of the type:
\[
\ddot{x} + 2\psi \omega_0 \dot{x} + \omega_0^2 x = \omega_0^2 \bar{x}
\]

with the initial conditions, which correspond to the system (1).

Here \( \psi, \omega_0 \) are the assigned factors of aperiodicity and frequency of the non-dissipated oscillations, which define the desirable motion properties of the object giving constant influence on its input.

Equation (8) can be presented in the form of (6), making a replacement:
\[
f(.) = \beta_0 (\bar{x} - x) - \beta_1 \dot{x}.
\]

Here \( \beta_0 = 1/\omega_0^2; \beta_1 = 2\psi \omega_0 \).

Apparently, factors \( \beta_0 \) also \( \beta_1 \) are unequivocally connected with the parameters \( \psi \) and \( \omega_0 \), which define the property of the reference motion of the amortized mass. Later on, the parameters \( \beta_0 \) and \( \beta_1 \) will be included into the control algorithms (regulators).

**Formulation of the problem of stability of motion**

The dynamic loading on a wheel \( P(t) \) is considered as the output variable. For the output variable \( P(t) \) the desirable motion properties are assigned in the following form:
\[
P = f_p(\bar{P}, \dot{P}, P),
\]

\( t \geq t_o; P(t_o) = P_o, \dot{P}(t_o) = \dot{P}_o \).

Here \( f_p(\cdot) \) is the assigned operator, generally nonlinear, defined on fuzzy sets; \( \bar{P} \) is the static loading.

And a condition of asymptotic stability of a system:
\[
P(t) \rightarrow \bar{P}(t), \dot{P}(t) \rightarrow 0 \text{ at } t \rightarrow \infty.
\]

It is required to construct a regulator, which provides fulfillment of the conditions (10) and (11).

In particular case, the desirable motion properties of the output variable are assigned by a linear equation of the following form:
\[
\ddot{P} + 2\psi \omega_0 \dot{P} + \omega_0^2 P = \omega_0^2 \bar{P}
\]

with the initial conditions, corresponding to the system (1).

Here \( \psi, \omega_0 \) are specified values of factors of aperiodicity and frequency of the non-dissipated oscillations of the transient for \( P(t) \).

For the formation of control for each of the mentioned problems it is necessary to define the corresponding phase coordinates. Hence, it is required to define the observable and identified variables for each problem. Both problems are realized on the same actuating mechanism.
Control Algorithms

Control for each of servo-drives is calculated using the following formula (indexes of supports and boards are left out):

$$u(\bar{t}, x, \ddot{x}, \dot{x}, x^{(4)}, f, \ddot{f}, \dot{f}, z, \ddot{z}, \dot{z}) =$$

$$\Phi_0 \left[ k_0 z + \left[ \Phi_1 (f, \ddot{f}, \dot{f}) + \Phi_2 (\bar{x}, x^{(4)}) + \Phi_3 (z, \ddot{z}, \dot{z}) \right] \right]$$ (12)

Here $\Phi_j (\cdot)$ are the known operators $(j = 0, \ldots, 3)$, $z$ is the output variable of the power actuating mechanism, $k_0$ is the constant, which describes the efficiency of the negative feedback on the output variable of the power actuating mechanism.

The spectrum of adaptive regulators can be obtained by variation of the functionals $\Phi_j$. As follows from the expression (12), from the point of view of calculations it is convenient to observe the accelerations of the controlled variables. Variables that are not observable, but are necessary for the purpose of control can be obtained by integration or differentiation.

The role of the functional $\Phi_0$ consists in giving desirable motion properties to the controlled system. The functional $\Phi_j$ reflects the actual condition of the object at each moment of time. The role of the functional $\Phi_j$ is reduced to the compensation of the imperfections of the actuating mechanism. The first member in the expression (12) compensates the attenuation of the control signal, if the feedback for the restriction of the output variable of the actuating mechanism is chosen a priori.

There are patents [14, 15] on the algorithm (12). In case of a special interest, the presented algorithm can be described completely. In the work [16], in addition, a method of control of the vehicle dynamics based on a new system with an active suspension are presented in the formulation of the general approach, we will consider two-mass systems with an active suspension. Here, for the simplicity of statements of control can be identically accurate within parameters or various on accuracy and speed.

Thus, parameters of resiliency are observed in an active suspension as well as parameters of dissipation. Energy for the accelerated adjustments of the functioning of the hydraulic cylinder is generated by means of an external source. The sensors of the wheel loading, of the displacement and of the acceleration transfer the signals to the electronic block of control within milliseconds. The control system allows reaching a constant loading on a wheel with maintenance of a constant average height of the vehicle. Steel springs or hydro- pneumatic elements of the suspension are used to maintain the static loading on a wheel.

Example of computer modeling

Mathematical models and program modules are realized for the multivariate systems of stabilization and active vibration protection. Computer modeling was made for the existing multi-wheel and caterpillar machines with an adaptive suspension. Here, for the simplicity of formulation of the general approach, we will consider two-mass systems with an active suspension undergoing stochastic kinematical perturbation.

The motion equations for a non-linear two-mass system with an active suspension are presented in the following form:

$$\dot{q}_1 = q_2, \quad \dot{q}_2 = -\Sigma_1 / m_1, \quad \dot{q}_3 = q_4, \quad \dot{q}_4 = (\Sigma_1 - \Sigma_2) / m_2, \quad q_5 = -(q_5 + k \cdot (u - k_0 q_6)) / T_5, \quad q_6 = -(q_6 + k \cdot q_5 - k_0 q_7) / T_5, \quad q_7 = -(q_7 + k \cdot q_8) / T_5, \quad q_8 = -(q_8 + k \cdot q_9) / T_5, \quad u(\bar{t}, x, \ddot{x}, \dot{x}, x^{(4)}, f, \ddot{f}, \dot{f}, z, \ddot{z}, \dot{z}) =$$

$$\Phi_0 \left[ k_0 z + \left[ \Phi_1 (f, \ddot{f}, \dot{f}) + \Phi_2 (\bar{x}, x^{(4)}) + \Phi_3 (z, \ddot{z}, \dot{z}) \right] \right]$$ (14)
Here $q_i$ are the phase coordinates ($i = 1, \ldots, 8$); $\sum_1, \sum_2$ are the "sheaves" of forces, generally non-linear, influencing accordingly on the masses $m_1$ and $m_2$:

$$\sum_1 = P_{11} + P_{21} + P_{31}$$
$$\sum_2 = P_{12} + P_{22} + P_{32}$$

$P_{11} = P_{11}(\Delta_1); P_{21} = P_{21}(\Delta_1); P_{31} = \bar{P}_{31} \sgn \Delta_1$

$P_{12} = P_{12}(\Delta_2); P_{22} = P_{22}(\Delta_2); P_{32} = \bar{P}_{32} \sgn \Delta_2$

$\Delta_1 = q_3 - q_1 + q_6; \quad \Delta_1 = q_4 - q_3 + q_6$

$\delta_2 = Q(t) - q_5; \quad \delta_2 = Q(t) - q_4$

where $P_{ij}(\cdot)$ are the known functions, generally non-linear; $\bar{P}_{31}, \bar{P}_{32}$ are the parameters of the dry friction in a static state in the suspension and in the tire, accordingly; $Q(t)$ kinematical perturbation; $Q(t) = dQ(t)/dt$. Plunger displacement of the actuating mechanism $q_6$ is included into the deformation of the elastic element $\Delta_1$; $k_z, T_z$ are accordingly the factor of amplification and the constant of time of the electro-hydraulic converter; $k_z, T_z$ are accordingly the factor of amplification and the constant of time of the power actuating mechanism (hydro-cylinder); $k_e$ is the parameter of the hydraulic engine, which describes outflows of the working body at its normal functioning; $k_u$ is the factor of the feedback by the position of the plunger of the actuating mechanism; $k_s, T_s$ are accordingly the factor of strengthening and the constant of time of the measuring device.

In a linear variant the elastic-dissipative forces look as follows:

$$P_{11} = C_1 \Delta_1; \quad P_{21} = K_1 \Delta_1; \quad P_{31} = 0$$
$$P_{12} = C_2 \Delta_2; \quad P_{22} = K_2 \Delta_2; \quad P_{32} = 0$$

Here $K_1, K_2$ are accordingly the factors of viscous dissipation of energy in the shock absorber and in the tire; $C_1, C_2$ are accordingly rigidities of the elastic element and the tire at static position.

The first two equations in the system (14) are equations of the control object (of the amortized mass $m_1$); the third and the fourth equations are the motion equations of the non-amortized mass $m_2$; the fifth is the equation of the electro-hydraulic converter (spool operated valve), the sixth is the equation of the power actuating mechanism (hydraulic cylinder); the seventh and the eighth are the equations of the sensors of accelerations of the masses $m_1$ and $m_2$. The other variables, which are used in the control algorithm, can be identified. The position sensor of the actuating mechanism is considered as non-inertial static part and, consequently, its equation is not examined.

When modeling the following restrictions are taken into account:

- on the displacement of the spool operated valve $q_6(t)$:
  $$q_{5\min} \leq q_6(t) \leq q_{5\max},$$
  where $q_{5\min}, q_{5\max}$ are the bottom and top permissible values;

- on the displacement of the variable $q_6(t)$ (the displacement of the plunger of the power hydro-cylinder):
  $$q_{6\min} \leq q_6(t) \leq q_{6\max},$$
  where $q_{6\min}, q_{6\max}$ are the bottom and top permissible values;

- on the control functions:
  $$u_\min \leq u(t) \leq u_\max,$$
  where $u_\min, u_\max$ are the bottom and top permissible values. In case if the variables $q_5, q_6$ and $u$ exceed the permissible limits, they put boundary values.

Research work was dedicated to five types of roads:

- concrete highway (type 1),
- cobble road (type 2),
- worn concrete highway (type 3),
- road with potholes (type 4),
- the broken dirt road (type 5).

For an estimation of efficiency of vibration protection root-mean-square values of vertical vibration accelerations for five octaves strips in frequencies range 0.88…22.4 Hz according to international standard ISO 2631-74 were used. For calculation root-mean-square values FFT algorithm with the subsequent recalculation of values of spectral density on octaves strips was used.

Numerical analysis was performed for five types of roads at movement of the wheel machine with speeds 5, 10, 15, 20 and 25 km/h with control system switched on and switched off.

On Figs. 1-5 distributions of root-mean-square magnitudes of vibration accelerations on octaves strips are shown:

- Solid style lines - correspond to a passive suspension bracket,
- Dashed-dotted lines – norms of ISO for various duration of influence of vibrations on human body,
- Dashed lines - an active suspension bracket.

Numbers 1-5 of curves correspond to speeds of movement: 1 - 5 km/h, 2 - 10 km/h, 3 - 15 km/h, 4 - 20 km/h, 5 - 25 km/h.

Vibration accelerations levels for road #1 are given in Fig. 1, for road #2 - in Fig. 2, for road #3 - in Fig. 3, for road #4 - in Fig. 4, for road #5 - in Fig. 5. The analysis of Figs. 1-5 indicates that for all types of roads at passive system vibration protection (control system is switched
off) vibration accelerations are beyond restrictions of ISO. At the same time with active vibration protection (control system is switched on)) vibration accelerations weights \( m_1 \) become insignificant on size for all types of roads, speed of movement practically does not influence a level of overloads if to be guided by 8-hour influence of vibrations on the person.

Introduction of a control system slightly increases the level of vibration accelerations in the fifth octave strip frequency, which implies insufficiently high speed of the selected electric-hydraulic drive. However the level of vibration accelerations in a strip of frequencies up to 8 Hz appears below a level 0.1 m/s² practically on all types of roads and on all high-speed modes. The obtained results confirm high efficiency of the control algorithm in a strip of frequencies of 0..9 Hz. By using faster operating electric-hydraulic drives, the upper frequency can be increased.

The results of the computer modeling of oscillations of the multi-wheel and caterpillar machines have shown high efficiency of the considered methods of the adaptive vibration protection and stabilization in a wide range of operating conditions.

![Fig. 1. Levels of vibration acceleration for road #1](image1)

![Fig. 2. Levels of vibration acceleration for road #2](image2)

![Fig. 3. Levels of vibration acceleration for road #3](image3)

![Fig. 4. Levels of vibration acceleration for road #4](image4)

![Fig. 5. Levels of vibration acceleration for road #5](image5)
References